MATHEMATICS 7

Shape and Space

Distance Learning

Alberta
EDUCATION

Module 6

Mathematics 7

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SHAPE AND SPACE

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Mathematics 7
Student Module Booklet
Module 6
Shape and Space
Alberta Distance Learning Centre
ISBN 0-7741-1244-1

This document is intended for	
Students	✓
Teachers (Mathematics 7)	✓
Administrators	
Parents	
General Public	
Other	

The Alberta Distance Learning Centre has an Internet site that you may find useful. The address is http://ednet.edc.gov.ab.ca/level3/adlc/300/adlchome.html.

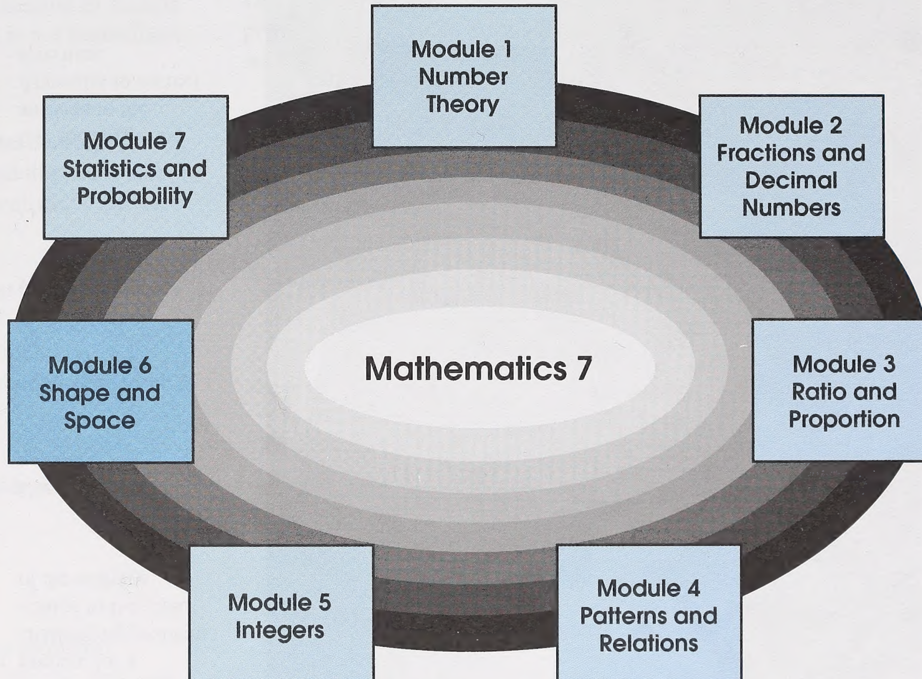
The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Welcome

WESTFILE INC.

Welcome to Module 6. We hope you'll enjoy your study of Shape and Space.

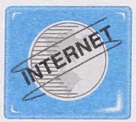
Mathematics 7 contains seven modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.



The document you are presently reading is called a Student Module Booklet. You may find visual cues or icons throughout it. Read the following explanations to discover what each icon prompts you to do.

- Prepare for a problem that will provide a change of topic.

- Prepare for a challenging problem related to the topic of the activity.



- Use the Internet to explore a topic.

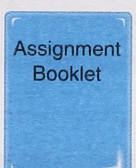
- Use computer software.

- Use a scientific calculator.

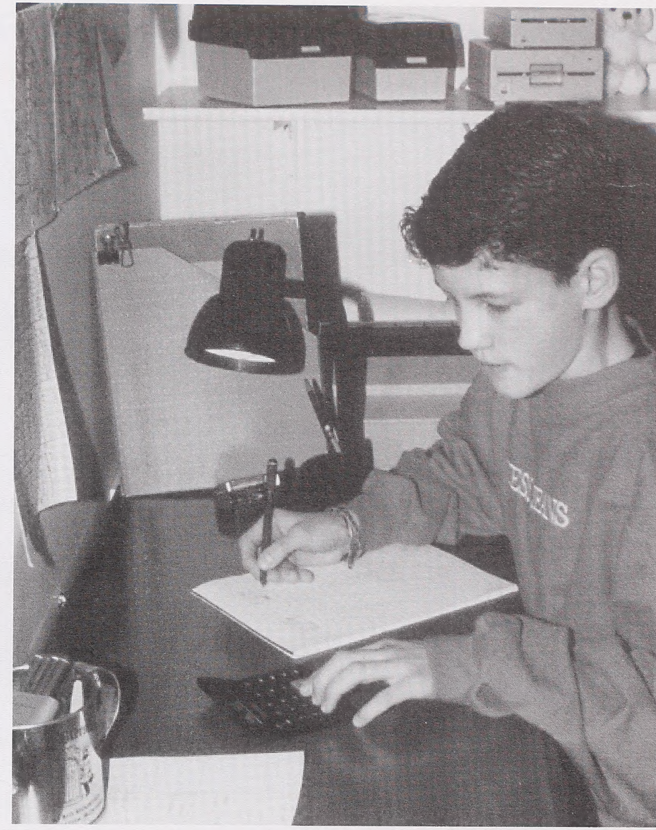
- View a videocassette.

- Pay close attention to important words or ideas.

- Use the suggested answers in the Appendix to correct activities.



- Answer the questions in the Assignment Booklet.



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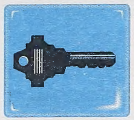
There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

Problem-Solving Skills

One of the exciting features of this course is that you will develop and improve your ability in problem solving. You will need these problem-solving skills many times in your lifetime. Since this course focuses on problem solving, it is important that you understand what a **problem** is.



A problem is a task for which the method of finding the answer (as well as the answer) is not immediately known.

Like any skill, the skill of problem solving must be developed. Problems may or may not involve computation (adding, subtracting, multiplying, and dividing). Some problems are realistic; others are puzzles.

You will have the opportunity in most activities to try a problem-solving challenge.

The  icon is a cue that the problem will be related to the topic of the activity.

The  icon is a cue that the problem will provide a change of topic.

The Four-stage Process

There are four stages that can be used to solve any problem: understanding the problem, developing a plan, trying the plan, and looking back.

Understanding the Problem

In this stage you should expect to feel puzzled. There are various reasons for feeling this way.

- You may not know the meanings of all the words.
- You may not understand the situation in the problem.
- You may be confused by unnecessary information.

Once you understand the problem, you should think about the problem and make an estimate of what the answer should be. This will help you arrive at a reasonable answer.

Developing a Plan

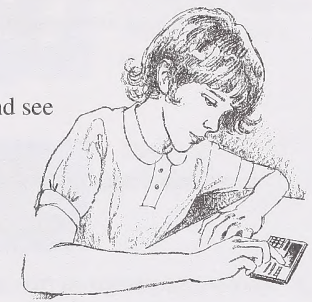
This is where you should decide on the plan of action that you are going to take to solve the problem.

You may consider the following strategies:

- changing your point of view
- using objects
- using diagrams
- making an organized list
- using Venn diagrams
- making a table
- guessing, checking, and revising
- acting out a problem
- working backwards
- simplifying a problem
- finding and applying a pattern
- using elimination
- using truth tables
- using an equation

Note: The Appendix in Module 1 explains these strategies in detail. When you see a problem-solving icon in any module, you should turn to the Appendix in Module 1 and review the problem-solving strategies.

Trying the Plan



In this stage you should try the plan and see if it works.

Be sure to work carefully and record your progress. You are encouraged to use a calculator to help with your calculations.

Note: While trying the plan, you should monitor your progress in order to determine if your plan will lead to a solution. You may find that the plan will not produce a solution, in which case a new plan will have to be developed.

Looking Back

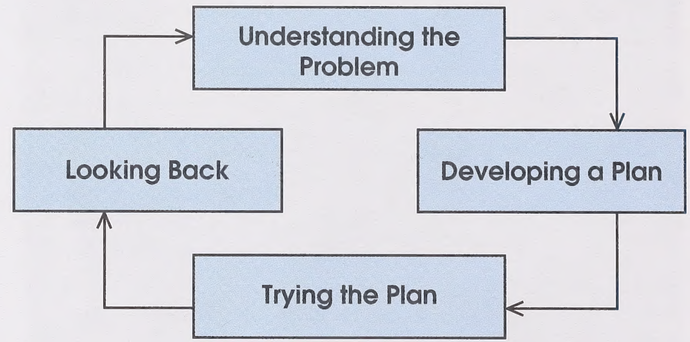
In this stage you should look back at the problem and compare your answer to the estimate you made in the first stage. Restate the problem using your answer.

Ask yourself these questions: "Did my plan work? Is my answer reasonable?"

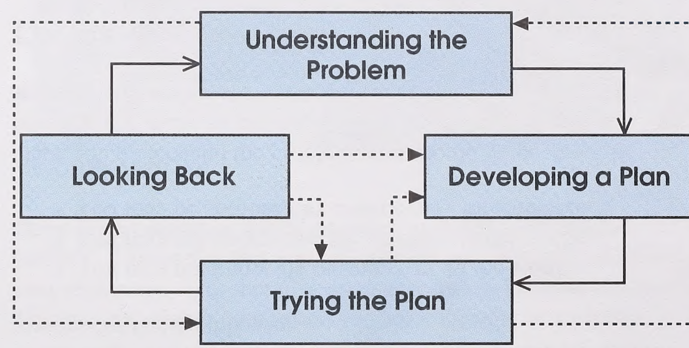
If you did not arrive at an answer, another strategy may work better. If your answer is unreasonable, you may have made errors while trying your plan.

Sequence of Stages

You usually approach a problem in the order outlined in the following diagram.



If you encounter difficulties in your original plan, or if you realize that another strategy will have better results, you may need to return to an earlier stage or use the stages in a different sequence.



CONTENTS

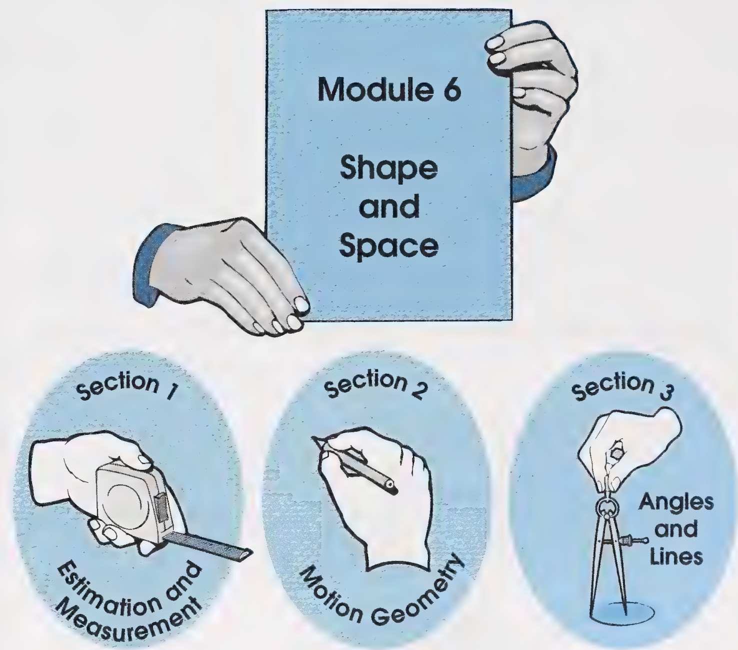
Module Overview

Do you enjoy working with your hands? Do you like to build things, use tools, and make designs? Do you agree with this old Chinese proverb?

I hear and I forget.
I see and I remember.
I do and I understand.

Most people enjoy hands-on activities and find learning easier if they become actively involved.

In Module 6, you will investigate shape and space; you will use maps, globes, and many different kinds of measuring tools and instruments. You will draw slide, flip, and turn images of figures. You will make designs by sliding, flipping, and/or turning a figure. You will construct angles and lines. You will sort and classify angles and lines.



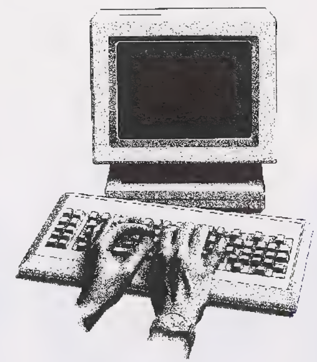
Evaluation

Your mark for this module will be determined by how well you complete the assignments at the end of each section and at the end of the module. In this module you must complete four assignments. The mark distribution is as follows:

Section 1 Assignment	**24 marks**
Section 2 Assignment	**22 marks**
Section 3 Assignment	**33 marks**
Final Module Assignment	**21 marks**
TOTAL	**100 marks**

When doing the assignments, work slowly and carefully. You must do each assignment independently, but if you are having difficulties, you may review the appropriate section in this module booklet.

If you are working on a CML terminal, you will have a module test as well as a module assignment.



Note

There is a final supervised test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

Section 1: Estimation and Measurement

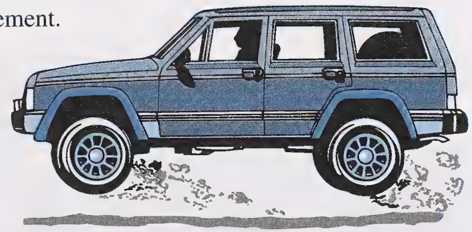
Have you watched the movie about the passenger jet that ran out of fuel and was forced to make an emergency landing at an old airstrip in Gimli, Manitoba? This movie is based on a true story. The jet did not have enough fuel for the journey because of an error calculating the amount of fuel required by the jet for the flight. Fortunately, no one was injured in this incident.

Estimation and measurement are important skills. Making errors can have costly results.

In this section you will estimate and measure length, mass, capacity, and angles. You will use scientific notation to describe large measurements. You will calculate the time and date changes that occur as you travel long distances and cross time zones. You will use maps, globes, and many different kinds of measuring tools and instruments.

Activity 1: Length, Mass, and Capacity

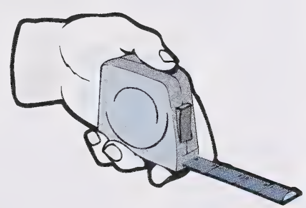
Everyone uses measurement.



You are probably very familiar with some measures. For example, you probably know your height and mass.

However, you may be less familiar with other measures. Do you know how many litres of gasoline your family vehicle holds? Do you know how long the vehicle is or how heavy it is?

Length



Finding the length, width, height, thickness, and circumference of things is so much a part of daily living that most people take it for granted.

1. If you did not have any measuring instruments and you did not know a system of measuring length, how would you measure the following lengths?

   a. the length of a paper clip
   b. the width of a desk
   c. the height of a room

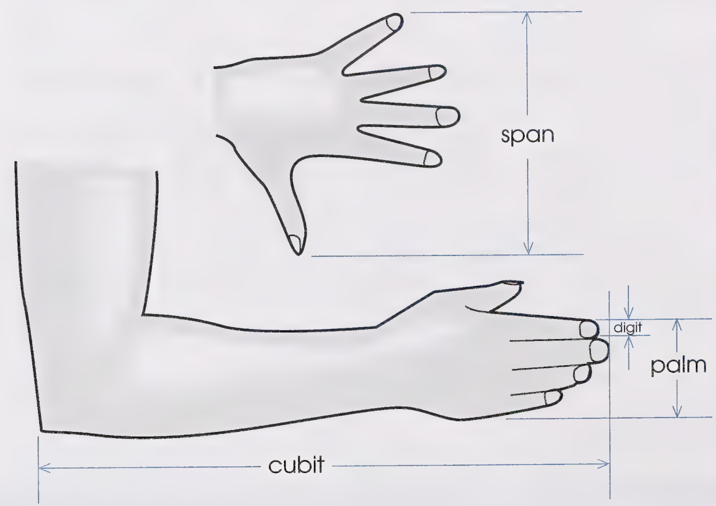
2. People today sometimes use informal language to describe lengths. What do each of the following phrases mean?

   a. a stone's throw
   b. seven paces wide

Check your answers by turning to the Appendix.

The ancient Egyptians, Hebrews, Greeks, and Romans all used the human body as a basis for their measurement systems.

For example, the ancient Hebrews measured lengths in digits (fingers), palms, spans, and cubits.



3. **a.** Use the old Hebrew units of length to measure the given dimension of each of the following objects. Ask your learning facilitator to also measure these objects. Then complete a chart like this.

Dimension of Object to be Measured	Your Measurement	Learning Facilitator's Measurement
length of a door in cubits		
length of a door in spans		
width of a calculator in palms		
width of a calculator in digits		

b. Is using body parts as units an accurate way to record measurements? Why or why not?

Check your answers by turning to the Appendix.

For many years Canadians used the **imperial system** of measurement. The basic unit of length in this system is the **yard**.

According to legend, the yard was originally equivalent to the distance between the nose and the fingertips of the outstretched arm of King Henry I of England (1100–1135).

Examine the following chart to discover how the other imperial units of length are related to the yard. (**Note:** You are not expected to remember this information.) Then answer question 4.

Imperial Units of Length
1 inch (in.) = $\frac{1}{36}$ yard
1 foot (ft.) = $\frac{1}{3}$ yard
1 yard (yd.)
1 rod = $5\frac{1}{2}$ yards
1 furlong = 220 yards
1 mile (mi.) = 1760 yards

4. Do you ever use any of these imperial units? If so, give the situation where you use the units.

Check your answer by turning to the Appendix.

In 1970, the **metric system** became Canada's official system of measurement. The basic unit of length in the metric system is the **metre**.

Examine the following chart to discover how the other metric units are related to the metre. Then answer question 5.

Metric Units of Length
1 millimetre (mm) = 0.001 metre
1 centimetre (cm) = 0.01 metre
1 decimetre (dm) = 0.1 metre
1 metre (m)
1 decametre (dam) = 10 metres
1 hectometre (hm) = 100 metres
1 kilometre (km) = 1000 metres

Note: 1 micrometre = 0.001 millimetre or 0.000 001 metre

5. **a.** What pattern do you notice in the chart?

b. Explain why the metric system is said to be easier to use than the imperial system.

c. Give reasons why many people in Canada continue to use the imperial system of measurement for some situations.

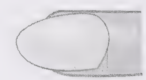
Check your answers by turning to the Appendix.

In the metric system, the most commonly used units for length are the millimetre, the centimetre, the metre, and the kilometre.

The following objects will help you get a sense for shorter lengths.

- The thickness of a dime is about 1 mm.

- The widest part of your little fingernail is about 1 cm wide.



- A doorknob is about 1 m from the floor.

Associating distances with time will help you get a sense for longer lengths.

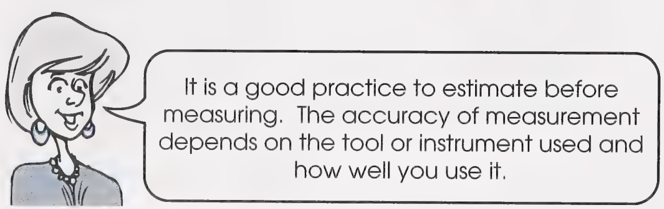
- At a brisk pace, an adult can walk about 1 km in 10 min.
- At a speed of 100 km/h, a car can travel about 100 km in 1 h.

6. Which unit would you use to describe the given dimension of each of the following objects?

   a. the width of a stamp
   b. the thickness of a toothpick
   c. the length of a key
   d. the height of a room from floor to ceiling

Check your answers by turning to the Appendix.

There are many different tools and instruments for measuring length; most have scales with subdivisions. When you measure, you must compare the object to the scale and find the closest subdivision.



Example 1

Use a metric ruler to find the length of this pencil. Give the measurement in centimetres and millimetres.

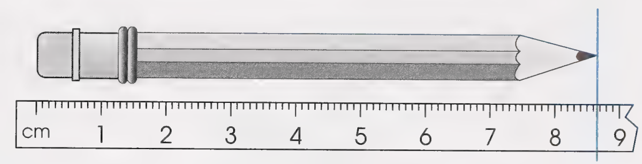
Solution

Step 1: Estimate the length of the pencil.

The pencil is about 9 widths of a person's little fingernail.

The pencil is about 9 cm long.

Step 2: Measure the pencil.

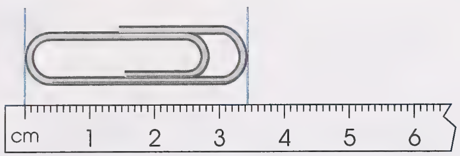


The pencil is about 8.7 cm or 87 mm long.

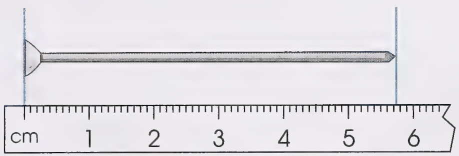
Note: The pencil is between 86 mm and 87 mm long. You could say the pencil is about 86.5 mm or 8.65 cm long.

7. Estimate and then measure each of the following objects in centimetres and in millimetres.

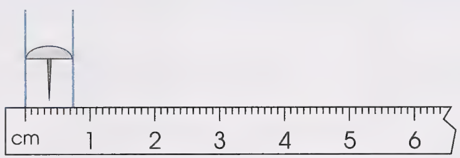
a.



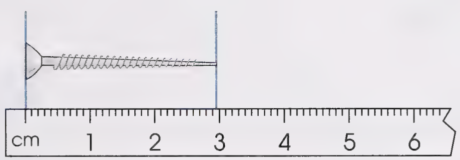
b.



c.

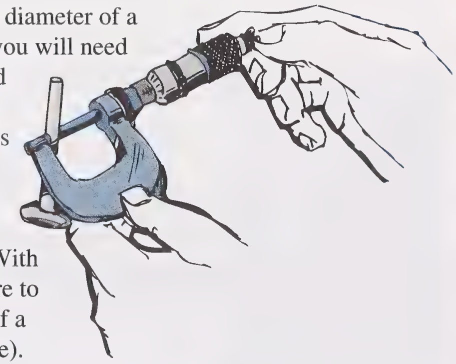


d.



Check your answers by turning to the Appendix.

You have seen some examples of items that can be measured using a metric ruler. There are other kinds of tools and instruments for measuring lengths. Here are two examples.



If you wish to check the diameter of a part on a small engine, you will need to make very careful and precise measurements; inaccurate measurements greatly affect the operation of the engine. You will probably use a **micrometre calliper**. With this tool you can measure to the nearest thousandth of a millimetre (a micrometre).

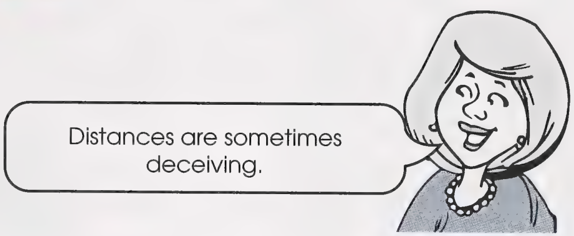
If you wish to measure the distance you travel on a bike, you do not need as precise a measurement. You will probably use an **odometer**. With this instrument, you can measure to the nearest kilometre or tenth of a kilometre.

8. Many tools and instruments are used to measure lengths—a metric ruler, a carpenter's measuring tape, a tailor's measuring tape, a pair of callipers, a depth gauge, an odometer, and so on.

What is the best tool or instrument to measure each of the following lengths?

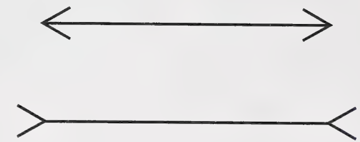
a. the length, width, and thickness of a book
b. the circumference of a small bottle
c. the length, width, and height of a room
d. the diameter of a pen
e. the distance a car travels

Check your answers by turning to the Appendix.



Example 2

Which of the following line segments is longer? Do not include the arrowheads in your estimate.



Solution

Actually, the line segments are the same length, but the bottom one may appear longer.

9. Estimate which length is longer in each of the following pairs.

   a. the length of your foot, the circumference of your ankle

   b. the length of your leg, the circumference of your waist

   c. the length of your foot, the distance from your wrist to your elbow

Check your answers by measuring each pair in question 9 and comparing the estimates to the measurements. (You will need a metric ruler, a tailor's metric measuring tape, and a carpenter's metric measuring tape.)

Mass

Now that you have reviewed estimating and measuring lengths, you are ready to estimate and measure mass.

The mass of an object is the amount of matter in the object.

In everyday language the word *weight* is often used instead of the word *mass*. Although weight and mass are different, the difference does not show up under ordinary conditions.

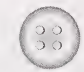
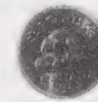
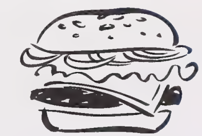
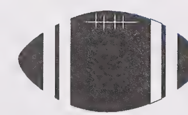
The basic unit of mass in the metric system is the **gram**. The following chart shows how the units are related.

Metric Units of Mass	Pattern
1 milligram (mg) = 0.001 gram	
1 centigram (cg) = 0.01 gram	×10
1 decigram (dg) = 0.1 gram	×10
1 gram (g)	×10
1 decagram (dag) = 10 grams	×10
1 hectogram (hg) = 100 grams	×10
1 kilogram (kg) = 1000 grams	×10

Note: 1 tonne (t) = 1000 kilograms
= 1 000 000 grams

In the metric system, the most commonly used units of mass are the milligram, the gram, and the kilogram.

The following objects will give you a sense of the size of these units.

- A straight pin has a mass of about 100 mg. 
- A small button, a raisin, and a small paper clip each have a mass of about 1 g.  
- A nickel has a mass of about 5 g. 
- A hamburger patty has a mass of about 100 g. 
- A football has a mass of about 1 kg. 

10. What unit would you use to describe the mass of each of the following items?

a. a whale
b. a cat
c. a person
d. a tropical fish
e. a pencil
f. an airplane
g. a penny
h. Earth
i. a hair

Check your answers by turning to the Appendix.

Estimating and measuring mass are important skills.

11. Visit the produce section of a supermarket and estimate the mass of different fruits and vegetables (such as the ones in the following list).

- a lemon
- a package of carrots
- a pea pod
- an avocado
- a head of lettuce
- a green pepper
- a bag of potatoes
- an apple
- a strawberry
- a bunch of bananas

Use a scale at the supermarket to check your estimates.

The scales at most supermarkets are easy to use. Some have dials like the one shown in the illustration. Others give digital readings.

However, if you have difficulty using the scale, the staff of the supermarket will be glad to help you.

12. Visit a school laboratory and estimate the masses of a variety of objects (such as the ones in the following list).

- a textbook
- a stapler
- a protractor
- a rubber band
- a tack
- a pair of scissors
- a shoe or runner
- a glue stick
- a pencil
- an eraser

Use a balance in the school laboratory to check your estimates.

There are many kinds of balances. The school laboratory you visit may have an **electronic balance**, a **one-pan balance**, or a **two-pan balance**. The student in the photograph is using a one-pan balance.

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It is easy to use an electronic balance; it gives a digital reading. Finding the mass with a pan balance requires more work. You will have to slide the weights along the beam(s) of the balance.

If you have not used a balance before, ask a teacher how to use one.

Capacity and Volume

Now that you have reviewed estimating and measuring mass you are ready to estimate and measure **capacity**.

The capacity of a container is the amount it will hold.

The basic unit of capacity in the metric system is the **litre**. The following chart shows how the units are related.

Metric Units of Capacity	Pattern
1 millilitre (mL) = 0.001 litre	×10
1 centilitre (cL) = 0.01 litre	×10
1 decilitre (dL) = 0.1 litre	×10
1 litre (L)	×10
1 decalitre (daL) = 10 litres	×10
1 hectolitre (hL) = 100 litres	×10
1 kilolitre (kL) = 1000 litres	

In the metric system, the most commonly used units of capacity are the millilitre and the litre. The following objects will help you get a sense of the size of these units.

- Milk is sold in 1 L, 2 L, and 4 L containers.
- A tablespoon holds about 15 mL.
- A teaspoon holds about 4 mL.

13. Which unit would you use to describe the capacity of each of the following?

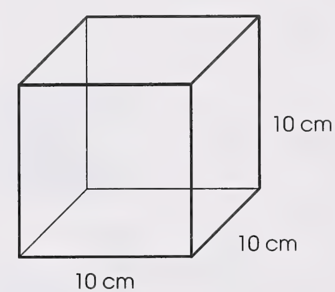
a. an eye dropper
b. a large paint can
c. a drinking glass
d. a mixing bowl

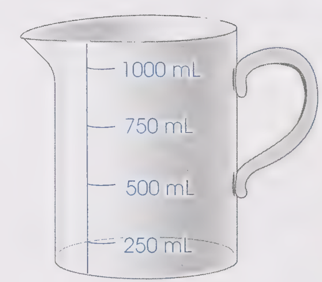
Check your answers by turning to the Appendix.

Now you will discover how capacity and **volume** are related.

Volume is the amount of space an object occupies.

For this investigation you will need a 1000-cm^3 cube, a 1000-mL measuring cup, and some rice or other dry ingredients.



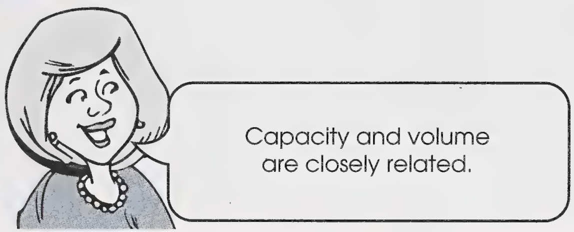


Note: If you do not have a 1000-cm^3 cube, you can make one by taping together five pieces of cardboard that are each 10 cm by 10 cm.

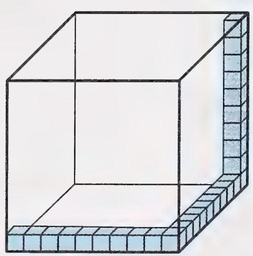
14. Fill the measuring cup to the 1000-mL mark with rice (or some other dry ingredient). Then empty the rice into the 1000-cm^3 cube.

How do the capacities of the measuring cup and the cube compare?

Check your answer by turning to the Appendix.



The capacity of the cube is 1000 mL. The cube occupies 1000 cm^3 of space.



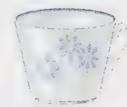
$$V = lwh$$
$$= 10 \times 10 \times 10$$
$$= 1000$$

So, 1000 mL of rice occupies 1000 cm^3.

This relationship is summarized in the following chart.

Capacity	Volume
1000 mL	1000 cm^3
1 mL	1 cm^3

15. Use the relationship between capacity and volume to complete the following.

a. 

350 mL = ______ cm^3

b. 

15 mL = ______ cm^3

c. 

2 L = ______ cm^3

d. 

3 L = ______ cm^3

Check your answers by turning to the Appendix.

There are different kinds of tools for measuring capacity. Metric cups and spoons are used to measure liquid ingredients (such as milk, water, and vinegar) and dry ingredients (such as flour, sugar, and spices).

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16. Gather the following empty containers. Then estimate the capacity of each of the containers.

a. a bottle cap	**b.** a drinking glass
c. a cereal box	**d.** a soup spoon
e. a shoebox	**f.** a soup bowl

Check your estimates by filling each container with rice (or some other dry ingredient), measuring the rice with a metric cup or spoon, and then comparing the estimates to the measurements.

Did You Know?

Are you being cheated when you find only half a box of cereal or half a bag of chips in a new package? No. Bulky products, such as cereals and chips, are packaged in large containers. However, they tend to settle over time. Such goods are packaged and sold by mass rather than volume.

Now Try This

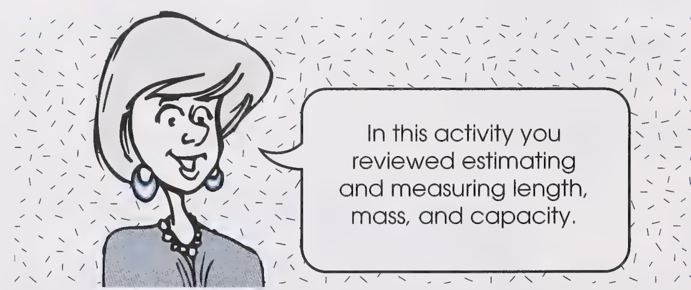
Use a problem-solving strategy to answer the following question.

17. Use the following rules to write a mathematical expression with a value of 3.

Rule 1: Each of the numbers 1, 2, 3, and 4 must be used at least once.

Rule 2: The symbols $+$, $-$, $\times$, $\div$, and ( ) may be used as needed.

Check your answer by turning to the Appendix.



Activity 2: Scientific Notation

With a diameter of 12 756.28 km, Earth is the fifth largest planet in the solar system. It has a volume of 1 083 230 000 000 km^3 and a mass of 5 976 000 000 000 000 000 000 t.

Measurements of large bodies such as Earth are usually not written in **standard form**. Instead, they are expressed in **scientific notation**.

Standard form is the usual form of a number. Scientific notation is a more compact form; the number is written as a number between 1 and 10, multiplied by a power of ten.

Example 1

As Earth orbits the Sun, it stays about 150 000 000 km from the Sun.

Write this measurement in scientific notation.

Solution

Step 1: Write the number 150 000 000. Use a caret to locate the decimal point for the number between 1 and 10.

$$1_{\wedge}50\ 000\ 000$$

So, the number between 1 and 10 is 1.5.

Step 2: Find the exponent of the power of ten by counting the digits that are to the right of the decimal point.

$$1_{\wedge}\underbrace{50\ 000\ 000}_{\text{8 digits}}$$

So, the exponent of the power of ten is 8.

Step 3: Write the number in scientific notation.

$$150\ 000\ 000 = 1.5 \times 10^8$$

Earth stays about 1.5×10^8 km from the Sun.

1. Write each of the following measurements in scientific notation.

   a. Earth has a diameter of 12 756.28 km.
   b. Earth has a volume of 1 083 230 000 000 km^3.
   c. Earth has a mass of 5 976 000 000 000 000 000 000 t.

Check your answers by turning to the Appendix.

When you encounter a number written in scientific notation, you may write it in standard form.

Example 2

The Moon is 3.84×10^5 km from Earth.

Write this measurement in standard form.

Solution

Step 1: Change the power of ten to its standard form.

$$10^5 = \underbrace{100\,000}_{5 \text{ zeros}}$$

Step 2: Multiply.

$$3.84 \times 10^5 = 3.84 \times 100\,000$$
$$= 384\,000$$

Multiplying a number by 10^5 or 100 000 moves the decimal point 5 places to the right.

So, the Moon is 384 000 km from Earth.

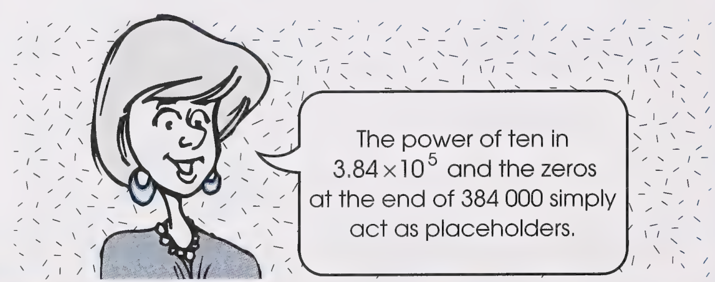
Note: This measurement is not very precise.

$3.\ 8\ 4 \times 10^5$

accurate accurate rounded

$3\ 8\ 4\ 0\ 0\ 0$

The measurement is rounded to the nearest thousand kilometres.



2. Write the following measurements in standard form.

   a. The temperature at the centre of the Sun is about 2.0×10^{7} °C.

   b. Earth is 1.496×10^{6} km from the Sun.

   c. Proxima Centauri, the nearest star outside the solar system, is about 4.00×10^{13} km from Earth.

Check your answers by turning to the Appendix.

Use the Internet to discover more about the solar system. You may find "Views of the Solar System" interesting; it is rated in the top 5% of all web sites. This is its uniform resource locator (URL):

http://bang.lanl.gov/solarsys/

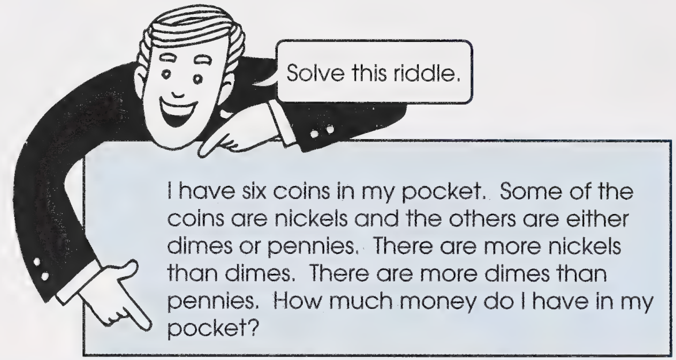
Note: This site gives information about the Sun, planets, moons, comets, and meteoroids found within the solar system. For example, you will discover the mass of the Sun is 1.989×10^{30} t (it is written as $1.989\text{e}+30$ on the Internet).

You may also wish to view some of the images and do some of the suggested activities.

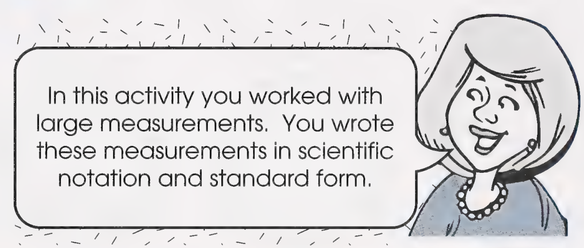
Now Try This

Use a problem-solving strategy to answer the following question.

3.



Check your answer by turning to the Appendix.



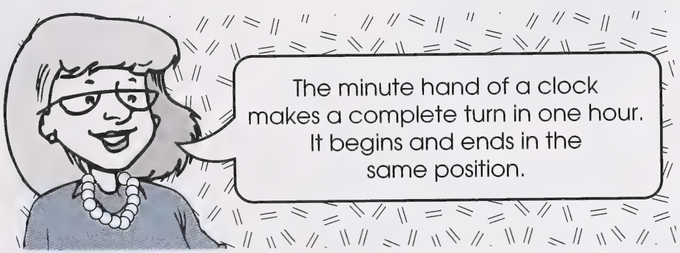
Activity 3: Angles

When you open a pair of scissors or a jackknife, you create an **angle**.

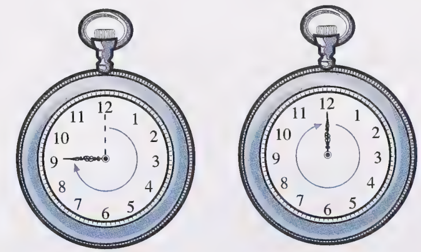
An angle is formed by two distinct **rays** starting from the same endpoint. A ray is a portion of a line starting at one point and going on forever.

In this activity, you will use turns and degrees to measure angles.

Turns

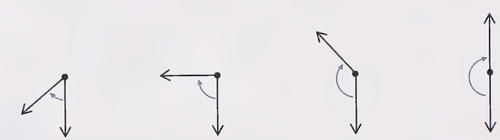


beginning position | position after one-quarter hour | position after one-half hour



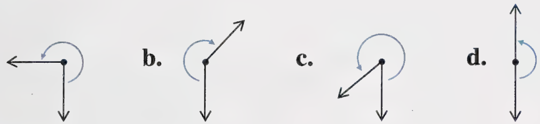
position after three-quarters of an hour | position after one hour

Angles can be measured in fractions of a turn. Here are some examples.



$\frac{1}{8}$ turn | $\frac{1}{4}$ turn | $\frac{3}{8}$ turn | $\frac{1}{2}$ turn

1. What is the measure, in fractions of a turn, of each of the following angles?

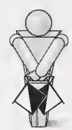


a. **b.** **c.** **d.**

Check your answers by turning to the Appendix.

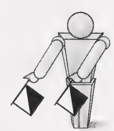
Semaphore is a way of sending messages using angles your arm makes with your body. Flags are usually used with these arm positions, but they are not necessary.

You begin with your arms in front of you.

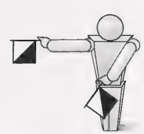


The first four letters of the alphabet are made with your right arm.

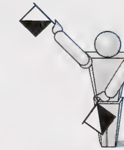
- To make **A**, you raise your right arm $\frac{1}{8}$ of a turn.



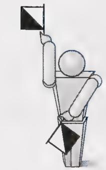
- To make **B**, you raise your right arm $\frac{1}{4}$ of a turn.



- To make **C**, you raise your right arm $\frac{3}{8}$ of a turn.

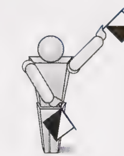


- To make **D**, you raise your right arm $\frac{1}{2}$ of a turn.

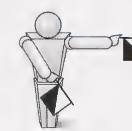


The next three letters are made with your left arm.

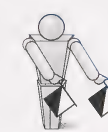
- To make **E**, you raise your left arm $\frac{3}{8}$ of a turn.



- To make **F**, you raise your left arm $\frac{1}{4}$ of a turn.

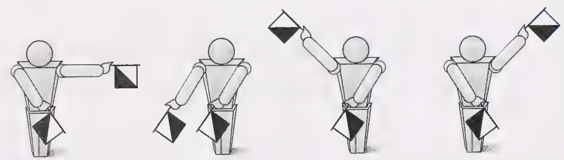


- To make **G**, you raise your left arm $\frac{1}{8}$ of a turn.



The rest of the letters are made with both arms.

2. What is the word signalled in this semaphore?



Check your answer by turning to the Appendix.

Degrees

The ancient Babylonians invented the notion of measuring angles in degrees. They determined that the Sun circled Earth every 365 days. They reasoned that the circle of the sky around Earth and every other circle should, therefore, be divided into 365 equal parts.

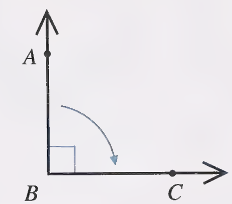
However, the Babylonians realized that 365 is a difficult number to work with because it has only the factors 1, 5, 73, and 365. The Babylonians chose to divide the circle into 360 parts instead. That was close enough, they reasoned, and more convenient to work with since 360 has more factors.

3. What are the factors of 360?

Check your answer by turning to the Appendix.

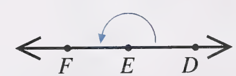
In order to estimate the measure of an angle in degrees, it is important to recognize a 90° angle and a 180° angle.

- A 90° angle is a quarter turn. $\angle ABC = 90°$



The symbol ∟ indicates an angle of 90°.

- A 180° angle is a half turn. $\angle DEF = 180°$



The symbol $\angle$ means "angle."

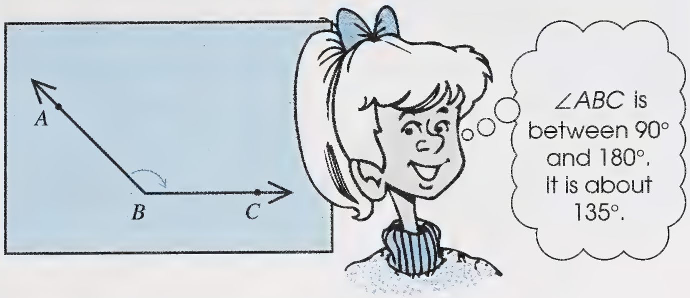
$\angle ABC$ is formed by the ray BA and the ray BC. The rays start from the endpoint B. This endpoint is called the **vertex** of the angle. The vertex of $\angle DEF$ is E.

Angles may be labelled in different ways. In the two examples, each angle was named by three letters, with the first letter standing for a point on one ray, the middle letter standing for the vertex, and the last letter standing for a point on the other ray.

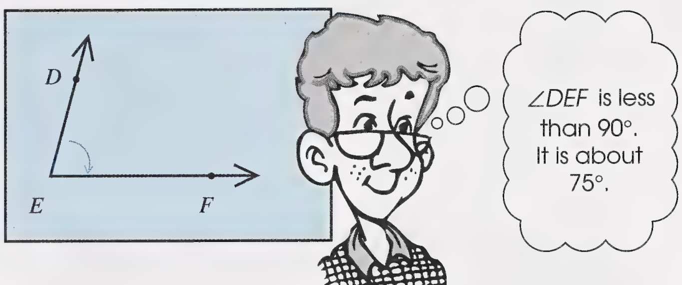
$\angle ABC$ could also be called $\angle CBA$; $\angle DEF$ could also be called $\angle FED$. Generally the point on the initial ray, where the turn begins, is given first.

You can estimate the measure of an angle in degrees by recognizing 90° angles and 180° angles.

Example 1

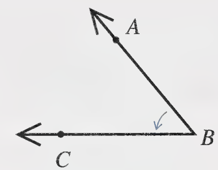


Example 2

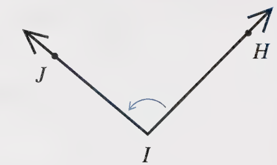


4. Estimate the measure of each of the following angles.

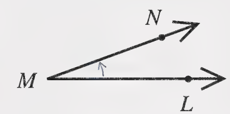
a.



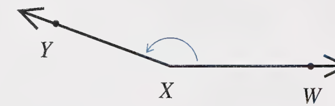
b.



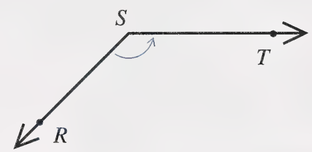
c.



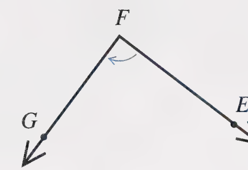
d.



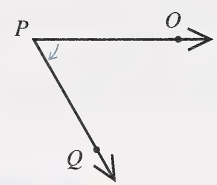
e.



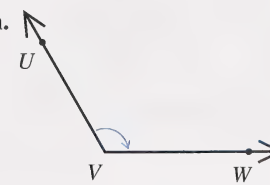
f.



g.



h.

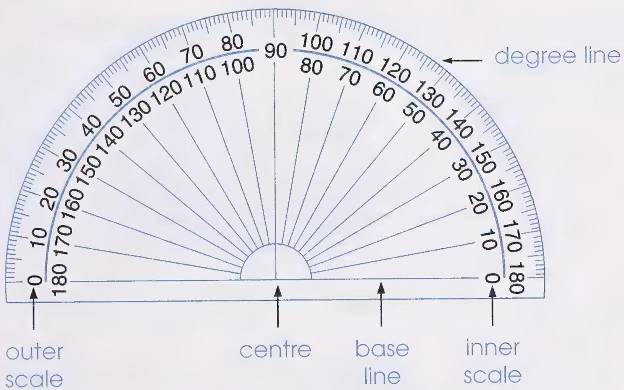


Check your answers by turning to the Appendix.

You can measure an angle in degrees with a semi-circular protractor.

A semi-circular protractor has the following parts:

- There are 180 **degree lines**.
- All the degree lines start at the **centre** of the protractor.
- The 0-degree or 180-degree line is called the **base line**.
- The protractor has an **outer scale** and an **inner scale**.



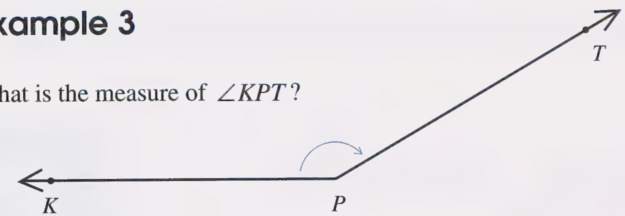
To measure an angle, you must do the following.

- Place the protractor on the angle. Be sure the base line of the protractor is on one ray of the angle, and the centre of the protractor is at the vertex of the angle.

- Use estimation to decide whether to read the inner or outer scale; then read the angle measure.

Example 3

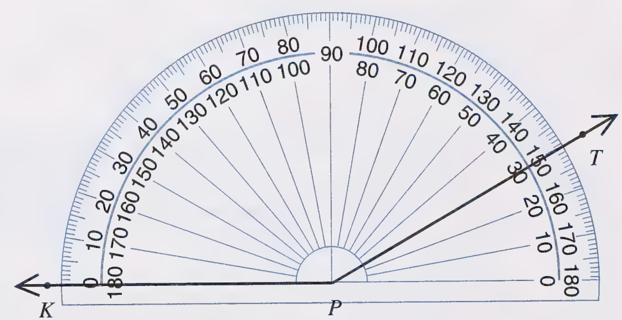
What is the measure of $\angle KPT$?



Solution

Step 1: Estimate the measure of $\angle KPT$. $\angle KPT$ is between 90° and 180°. It is about 135°.

Step 2: Measure $\angle KPT$. Place the protractor on the angle.



Step 3: Use the estimation to decide whether the correct measure is 30° or 150°.

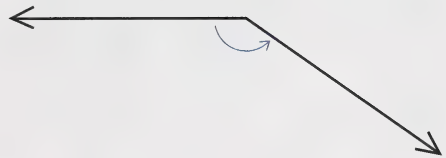
$\angle KPT$ is between 90° and 180°.

$\therefore\ \angle KPT = 150°$

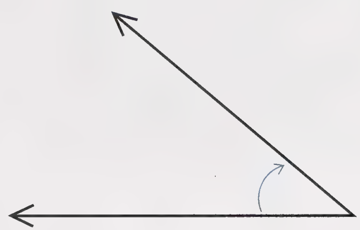
5. Measure each of the following angles with a semi-circular protractor.

a.

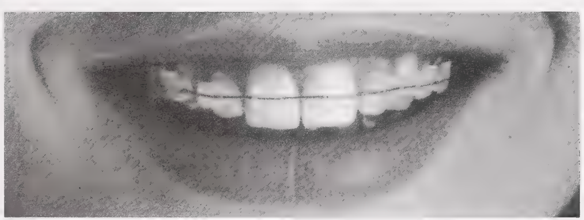
b.



c.



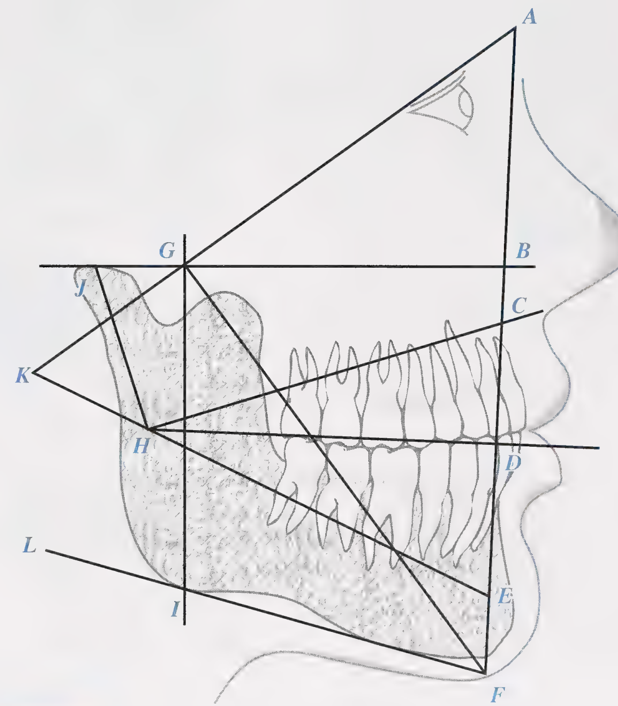
6. Dentists make angles on prints of dental x-rays. They measure these angles to find out the patient's face type before braces are applied.



NASA

Measure each of these angles on the following diagram.

a. $\angle KGF$ b. $\angle GBF$ c. $\angle CHD$
d. $\angle CHE$ e. $\angle JHD$ f. $\angle AKE$



Check your answers by turning to the Appendix.

Did You Know?

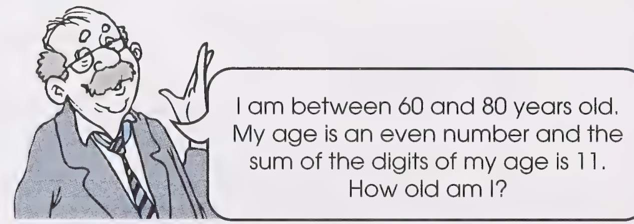
It is easy to make an error of 1° when you are measuring angles with a protractor, so you may think that an error of 1° is not very important. However, in some situations an error of 1° can be very costly. Here is an example.

An airplane is flying from Montreal to Vancouver, a distance of 3235 km. If the plane is 1° off course, it could end up 55 km from the Vancouver airport.

Now Try This

Use a problem-solving strategy to answer the following question.

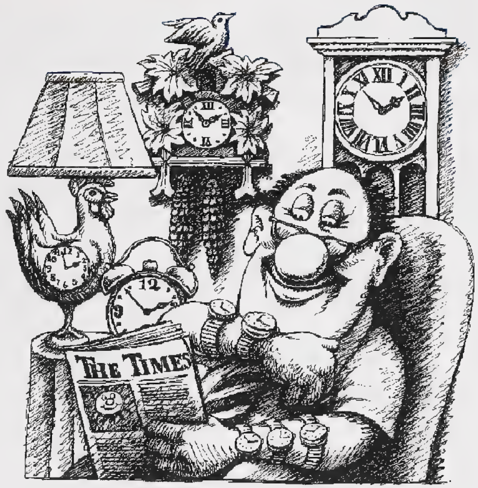
7.



What is the speaker's age?

Check your answer by turning to the Appendix.

Activity 4: Time



Time is an important part of daily life. No one wants to be late for an appointment or miss a favourite television show. However, giving the time can be confusing.

For example, when people say, “We’ll telephone you at 8 o’clock,” do they mean 8 o’clock in the morning or 8 o’clock in the evening? If the people live in another part of Canada, do they mean 8 o’clock your time or 8 o’clock their time?

In this activity you will use the 24-hour clock and solve problems involving time zones.

The 24-hour Clock

To avoid confusion, time can be given using the 24-hour clock.

Example 1

Using the 24-hour clock, 8:00 A.M. is written as 08:00 (read as “eight” or “eight zero”) and 8:00 P.M. is written as 20:00 (read as “twenty” or “twenty zero”).

Example 2

Using the 24-hour clock, 3:30 P.M. is written as 15:30 (read as “fifteen thirty”).

1. Write each of the following times using the 24-hour clock.

a. 9:00 A.M. **b.** noon
c. 3:15 P.M. **d.** 10:20 P.M.

2. Write each of these times using A.M. or P.M.

a. 07:00 **b.** 13:20
c. 16:40 **d.** 23:00

Check your answers by turning to the Appendix.

Canadian Time Zones

Why are there time zones?

To answer this question, view the video *Why We Have Time Zones*; then read the following explanation of how time zones were invented.

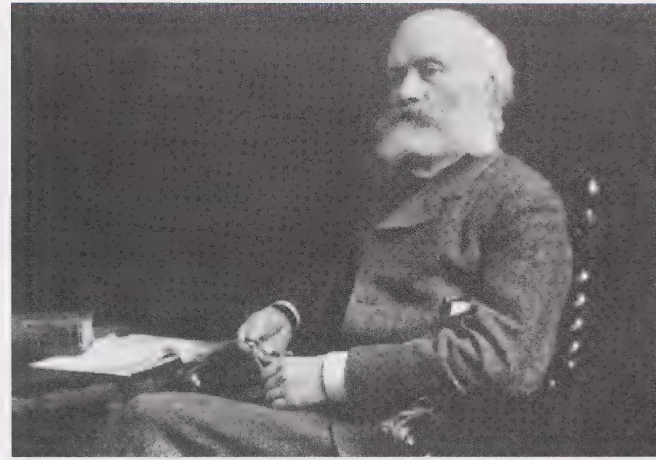
Before there were time zones, **solar time** was used.

Solar time is a system of keeping time. In this system, noon is considered to be the time when the sun is directly overhead. With the solar time system, time varies from place to place according to the position of the sun.

Using solar time was satisfactory as long as people did not travel far from their own community. However, with the construction of the railways in North America, people had the opportunity to travel longer distances more easily, and the system of solar time created problems for railway companies. For scheduling purposes, a conductor on a train had to keep dozens of clocks, each set to the time of one of the towns on the route.

In 1878, Sir Sanford Fleming (a Canadian) proposed a system of telling time that would end the confusion caused by the use of solar time. The system was called **standard time**.

At the time of his proposal, Fleming was the chief engineer for the Canadian Pacific Railway and he was about to complete the first trans-Canada railroad. Fleming is shown in the following photograph.



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Fleming made the following recommendations.

- Canada and the United States should be divided into several time zones: the Atlantic, Eastern, Central, Mountain, and Pacific.
- The time should be the same for all places within each time zone.
- The time zones should differ from those on either side by one hour.

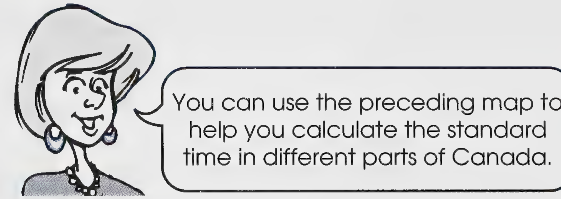
Fleming's ideas were favourably received, and by 1883, all the railways in North America were using this system of keeping time.

Examine the following map; it shows the time zones in Canada today. Notice that when it is 01:00 Pacific Standard Time (PST), it is 02:00 Mountain Standard Time (MST), 03:00 Central Standard Time (CST), 04:00 Eastern Standard Time (EST), 05:00 Atlantic Standard Time (AST), and 05:30 Newfoundland Standard Time (NST).

Standard Time Zones in Canada

SOURCE OF MAP DATA: STATISTICS CANADA

Note: This map and the following examples do **not** deal with time during the period of **daylight-saving time**. There is information on daylight-saving time in the Enrichment.



Example 1

Mhairi Murphy, who lives in Ottawa, wants to call a friend in Vancouver. She looks at her watch and notices that it is 8 P.M. or 20:00 EST. Give the time in Vancouver when it is 20:00 EST in Ottawa.

Solution

Step 1: Use the map to find the change in time (time difference) from the Eastern time zone to the Pacific time zone.

04:00 EST = 01:00 PST

So, the time in the Pacific zone is 3 hours **less** than the time in the Eastern zone.

Step 2: Calculate the time in the Pacific zone.

$$\begin{array}{r} 20:00 \\ -\quad 3 \\ \hline 17:00 \end{array}$$

The time in Vancouver is 3 h less than the time in Ottawa, so subtract 3 h.

When it is 20:00 EST in Ottawa, it is 17:00 PST in Vancouver.

Example 2

Mr. Ruhl, who lives in Calgary, wants to call a business in Toronto. He looks at his watch and notices that it is 8 A.M. or 08:00 MST. Give the Eastern standard time when it is 08:00 MST.

Solution

Step 1: Use the map to find the change in time (time difference) from the Mountain time zone to the Eastern time zone.

02:00 MST = 04:00 EST

So, the time in the Eastern zone is 2 h **more** than the time in the Mountain zone.

Step 2: Calculate the time in the Eastern time zone.

$$\begin{array}{r} 08:00 \\ +\quad 2 \\ \hline 10:00 \end{array}$$

The time in Toronto is 2 h more than the time in Calgary, so add 2 h.

When it is 08:00 MST in Calgary, it is 10:00 EST in Toronto.

Use the map entitled "Standard Time Zones in Canada" to answer the following questions.

3. If it is 10:00 MST, what time is it in each of the following standard time zones?

   a. Atlantic **b.** Central

4. If it is 22:00 AST, what time is it in each of the following standard time zones?

   a. Mountain **b.** Eastern

Check your answers by turning to the Appendix.

Did You Know?

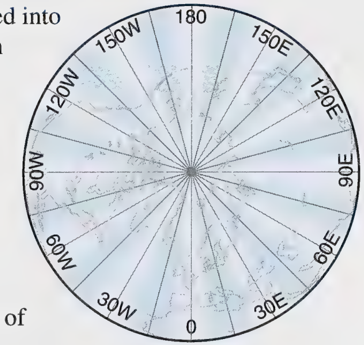
Newfoundland's west coast almost reaches the Atlantic time zone but the most populated region is on the east coast. Newfoundland could have decided to use Atlantic Standard Time or it could have stayed one hour ahead. The province decided to split the difference, and it now has its own time zone. Newfoundland Standard Time is one-half hour ahead of Atlantic Standard Time.

International Time Zones

Because of the efforts of Sir Sanford Fleming, standard time spread throughout the world.

By 1883, the railways in North America were using standard time. In 1884, Fleming spoke to a conference with delegates from twenty-seven countries and shared his ideas for an international system of standard time.

Fleming proposed the following:



- The world should be divided into 24 time zones, one for each hour of the day.
- Each time zone should be centred around an imaginary line called a **meridian**.
- The meridians should run north and south along lines of longitude.
- The first meridian, the **prime meridian**, should run along 0° longitude and go through Greenwich, England.
- The other meridians should be 15° apart $(360 \div 24 = 15)$.

Fleming's ideas were basically accepted. Considerations, however, were made for various borders.

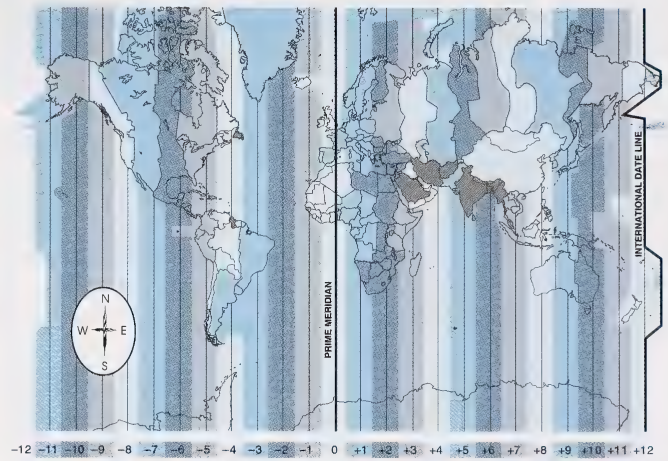
Examine the map entitled "International Standard Time Zones" in the Appendix. Notice the integers at the bottom of the map. Each integer shows the time difference, in hours, from the time on the prime meridian. You can use this map to help you find the time in any part of the world if you are given the time on the prime meridian.

Example 1

When it is 08:00 on the prime meridian, what time is it in Winnipeg?

Solution

Step 1: Find the time zone in which Winnipeg is located. Give the corresponding integer at the bottom of the map.



Step 2: Calculate the time in Winnipeg.

$$\begin{array}{r} 08:00 \\ -\quad 6 \\ \hline 02:00 \end{array}$$

The time in Winnipeg is 6 h **less** than the time at the prime meridian, so **subtract** 6 h.

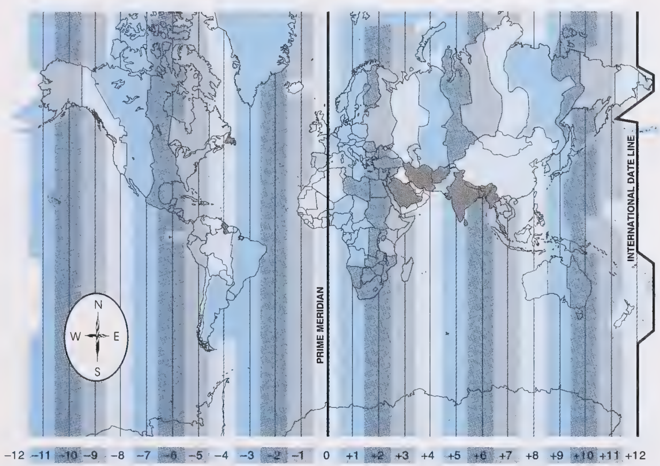
When it is 08:00 on the prime meridian, it is 02:00 in Winnipeg.

Example 2

When it is 13:00 on the prime meridian, what time is it in Beijing, China?

Solution

Step 1: Find the time zone in which Beijing is located. Give the corresponding integer at the bottom of the map.



Beijing is in this time zone; the corresponding integer is +8.

Step 2: Calculate the time in Beijing.

$$\begin{array}{r} 13:00 \\ +\quad 8 \\ \hline 21:00 \end{array}$$

The time in Beijing is 8 h **more** than the time at the prime meridian, so **add** 8 h.

When it is 13:00 on the prime meridian, it is 21:00 in Beijing.

Use the map entitled "International Standard Time Zones" in the Appendix to answer the following questions.

5. When it is 20:00 on the prime meridian, what is the time in each of the following communities in Canada?

a. Victoria **b.** Edmonton **c.** Regina
d. Winnipeg **e.** Toronto **f.** Quebec City
g. Fredericton **h.** Charlottetown **i.** Halifax
j. Whitehorse **k.** Yellowknife **l.** St. John's

6. When it is 06:00 on the prime meridian, what time is it in each of these international cities? **Hint:** You may first need to use an atlas or a globe to locate these cities.

a. Melbourne, Australia **b.** Lima, Peru
c. Tokyo, Japan **d.** Cape Town, South Africa
e. Rome, Italy **f.** New York, U.S.A.

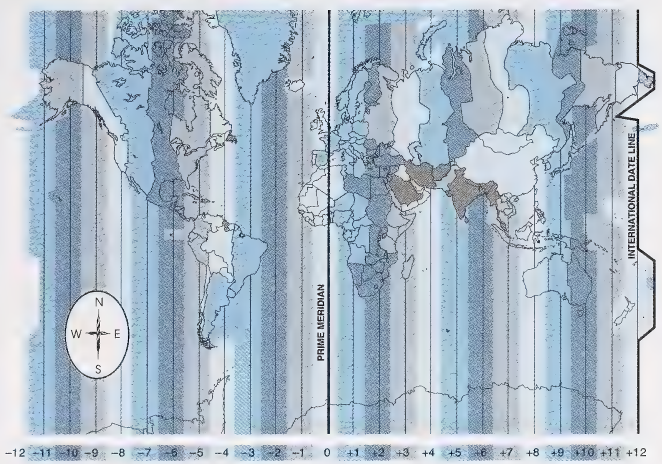
Check your answers by turning to the Appendix.

You are now ready to calculate the time in a time zone given the time in another time zone.

Example 3

When it is 05:00 in Edmonton, Canada, what time is it in Wellington, New Zealand?

Step 1: Find the time zones in which Edmonton and Wellington are located. Give the corresponding integers at the bottom of the map.



Edmonton is in this time zone; the corresponding integer is -7.

Wellington is in this time zone; the corresponding integer is +12.

Step 2: Calculate the change in time from Edmonton to Wellington.

$$\begin{aligned}(+12)-(-7) &= (+12)+(+7)\\ &= +19\end{aligned}$$

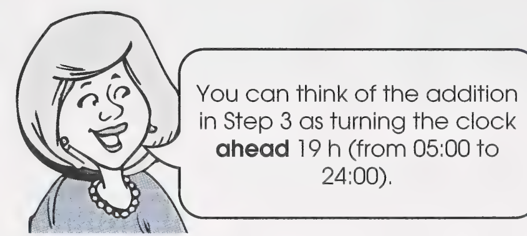
The time in Wellington is 19 h **more** than the time in Edmonton.

Step 3: Calculate the time in Wellington.

$$\begin{array}{r} 05:00 \\ +\ 19 \\ \hline 24:00 \end{array}$$

When it is 05:00 in Edmonton, it is 24:00 in Wellington.

Note: You can check your calculations in Step 2 by counting the number of time zones from Edmonton to Wellington. Counting in an **eastward** direction, you cross 19 time zones. Because you counted in an eastward direction, the time difference is **positive**. So, the time in Wellington is 19 h **more** than the time in Edmonton.

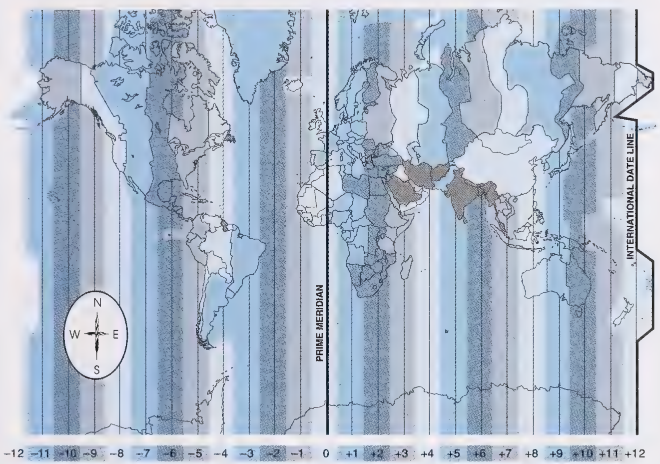


Example 4

When it is 17:00 in Paris, France, what time is it in Whitehorse, Canada?

Solution

Step 1: Find the time zones in which Paris and Whitehorse are located. Give the corresponding integers at the bottom of the map.



Whitehorse is in this time zone; the corresponding integer is -8.

Paris is in this time zone; the corresponding integer is +1.

Step 2: Calculate the change in time from Paris to Whitehorse.

$$\begin{aligned}(-8)-(+1)&=(-8)+(-1)\\&=-9\end{aligned}$$

The time in Whitehorse is 9 h **less** than the time in Paris.

Step 3: Calculate the time in Whitehorse.

$$\begin{array}{r}17:00\\-\quad 9\\\hline 08:00\end{array}$$

When it is 17:00 in Paris, it is 08:00 in Whitehorse.

Note: You can check your calculations in Step 2 by counting the number of time zones from Paris to Whitehorse. Counting in a **westward** direction, you cross 9 time zones. Because you counted in a westward direction, the time difference is **negative**. So, the time in Whitehorse is 9 h **less** than the time in Paris.

You can think of the subtraction in Step 3 as turning the clock **back** 9 h (from 17:00 to 08:00).

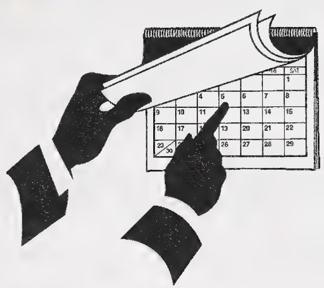
Use the map entitled "International Standard Time Zones" in the Appendix to answer the following question.

7. When it is 10:00 in Ottawa, what time is it in each of the following countries?

a. Argentina **b.** Scotland **c.** Italy
d. China **e.** Japan **f.** South Africa

Check your answers by turning to the Appendix.

Date Changes



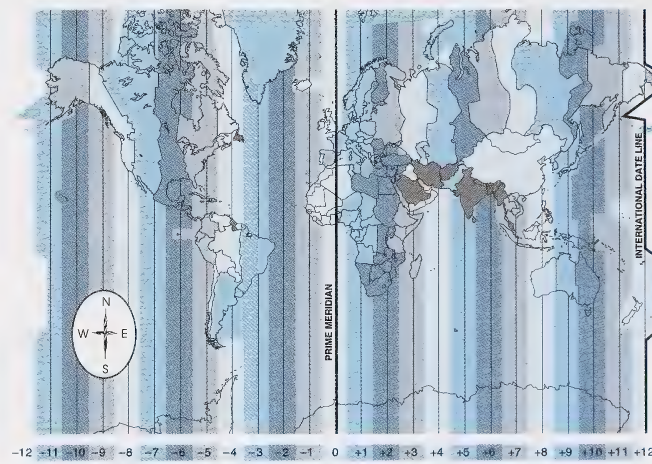
The video *Why We Have Time Zones*, which you watched earlier in this activity, shows what happens as the world rotates—different parts of the world experience day and night. A new day begins in every time zone after the clock reaches 24:00 (midnight). Therefore, you must consider date changes when you calculate time.

Example 1

It is 22:00 on Thursday in Calgary. What time and day is it in Halifax?

Solution

Step 1: Find the time zones in which Calgary and Halifax are located. Give the corresponding integers at the bottom of the map.



Step 2: Calculate the change in time from Calgary to Halifax.

$$\begin{aligned}(-4)-(-7) &= (-4)+(+7)\\ &= +3\end{aligned}$$

The time in Halifax is 3 h **more** than the time in Calgary.

Step 3: Calculate the time and day in Halifax.

$$\begin{array}{r} 22:00 \\ +\quad 3 \\ \hline 25:00 = 01:00 \end{array}$$

Regrouping is required; 25:00 on Thursday is the same as 01:00 on Friday.

The time and day in Halifax is 01:00 on Friday.

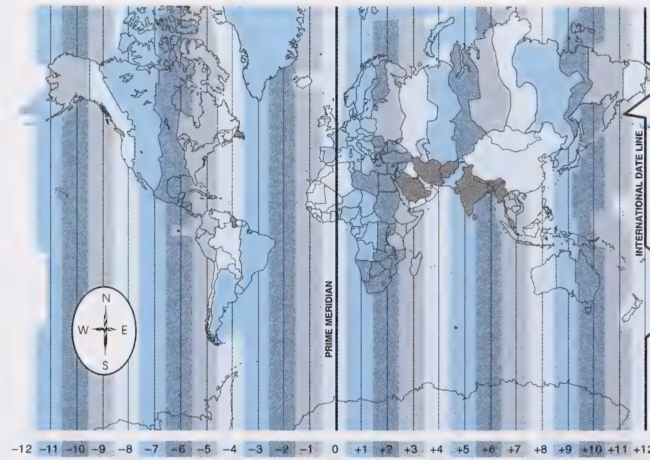
You can think of the **addition** in Step 3 as turning the clock **ahead** 3 h (from 22:00 to 01:00). Because you turned the clock ahead past midnight, it is the next day.

Example 2

It is 02:00 on Monday in Paris, France. What time and day is it in Edmonton, Canada?

Solution

Step 1: Find the time zones in which Paris and Edmonton are located. Give the corresponding integers at the bottom of the map.



Edmonton is in this time zone; the corresponding integer is -7.

Paris is in this time zone; the corresponding integer is +1.

Step 2: Calculate the change in time from Paris to Edmonton.

$$\begin{aligned}(-7)-(+1) &= (-7)+(-1)\\ &= -8\end{aligned}$$

The time in Edmonton is 8 h **less** than the time in Paris.

Step 3: Find the time and day in Edmonton.

$$\begin{array}{r} 02:00 \\ -\quad 8 \\ \hline \end{array} = \begin{array}{r} 26:00 \\ -\quad 8 \\ \hline 18:00 \end{array}$$

Regrouping is required; 02:00 on Monday is the same as 26:00 on Sunday.

It is 18:00 on Sunday in Edmonton.

Note: You can think of the **subtraction** in Step 3 as turning the clock **back** 8 h (from 02:00 to 18:00). Because you turned the clock back past midnight, it is the day before.

Use the map entitled "International Standard Time Zones" in the Appendix to answer the following questions.

8. a. If it is 23:00 on Monday in Regina, what time and day is it in Charlottetown?

b. If it is 01:00 on Thursday in Ottawa, what time and day is it in Vancouver?

c. If it is 07:30 on Wednesday in Tokyo, Japan, what time and day is it in Winnipeg, Canada?

d. If it is 18:15 on Friday in Whitehorse, Canada, what time and day is it in Melbourne, Australia?

Check your answers by turning to the Appendix.

The **International Date Line** is also important when considering date changes.

The International Date Line is an imaginary line that runs north and south, mostly along 180° longitude.

9. a. On a globe, find the International Date Line. **Note:** Most schools and libraries have globes. Many homes also have globes.

Next, find the International Date Line(s) on the map entitled "International Standard Time Zones" in the Appendix.

Explain why the date line is shown as **one** line on the globe, but is shown as **two** lines on the map.

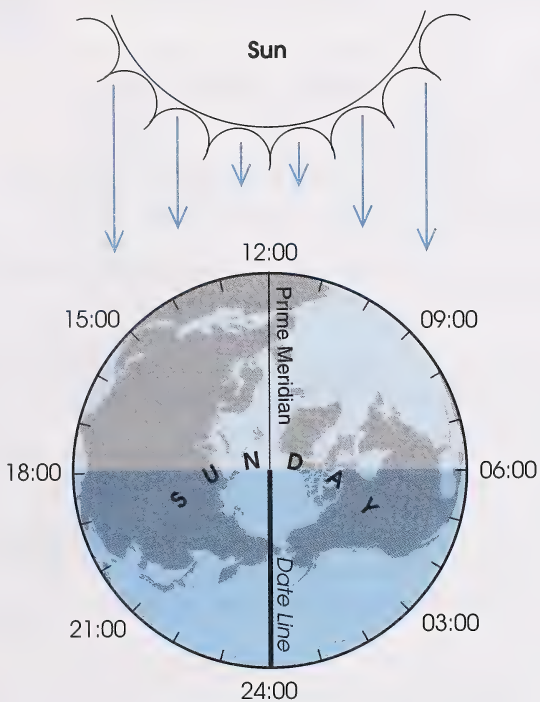
b. On a globe, find the prime meridian. Where is the date line in relation to the prime meridian?

Check your answers by turning to the Appendix.

Study figures 1 to 4 to discover how the date changes across the world in relation to the date line. **Note:** In the figures, the date line is shown as a straight line for simplicity sake; in reality it zigzags.

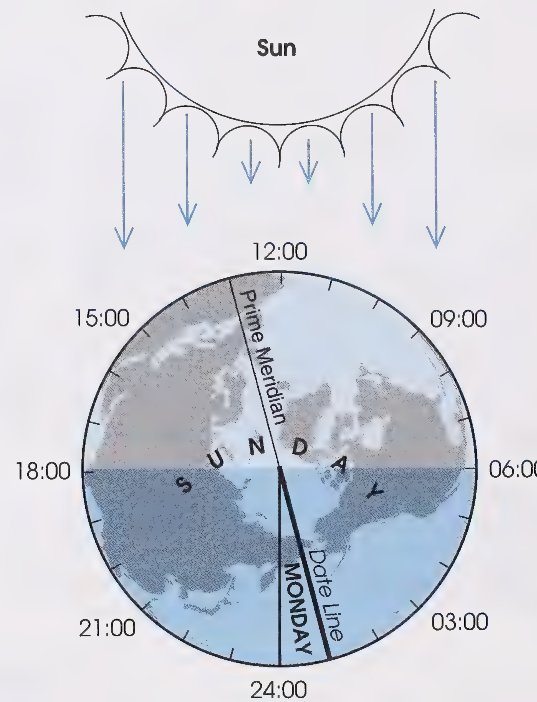
Figure 1 shows that it is Sunday all over the world. The shading on the map shows that it is night in the lower portion of the world and day in the upper portion of the world. It is midnight on the date line; it is noon on the prime meridian.

Figure 1



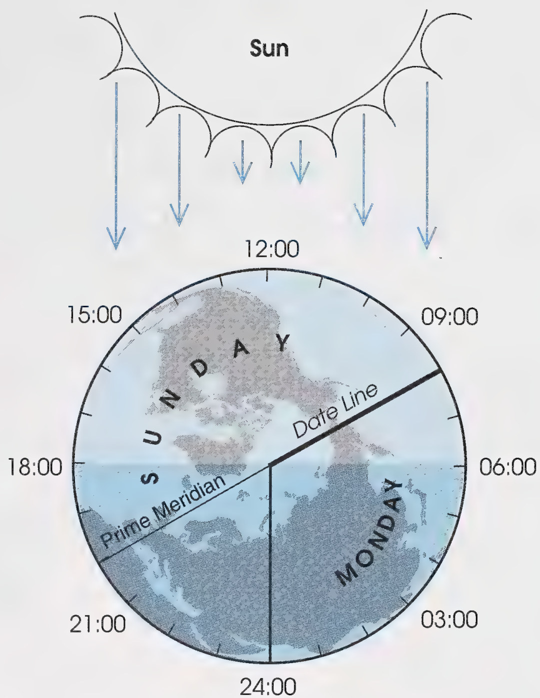
In Figure 2, it is 1 h later than in Figure 1. The world has rotated 15° and the area that is Monday appears as a thin wedge to the west of the date line. On the date line it is 01:00 on Monday; on the prime meridian it is 13:00 on Sunday.

Figure 2



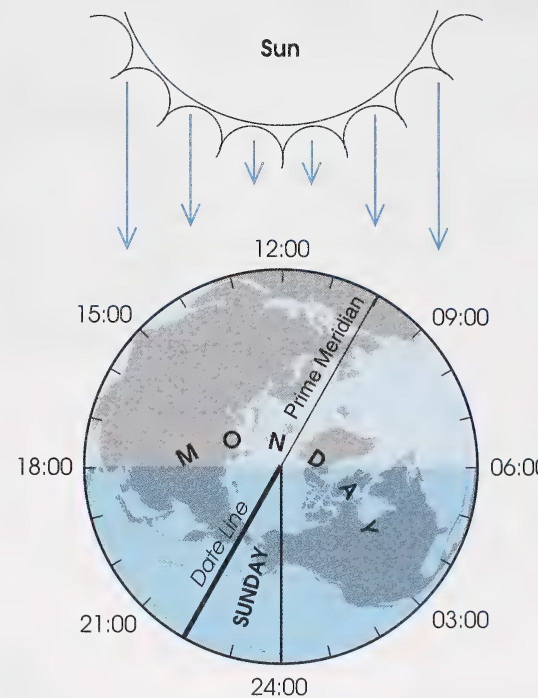
In Figure 3, it is 8 h later than in Figure 1. The world has rotated 120° and the area that is Monday appears as a wide wedge to the west of the date line. On the date line it is 08:00 on Monday; on the prime meridian it is 20:00 on Sunday.

Figure 3



In Figure 4, it is 22 h later than in Figure 1. The world has rotated 300°. The area that is Sunday is now only a thin wedge and it is Monday over most of the world. On the date line it is 22:00 on Monday; on the prime meridian it is 10:00 on Monday.

Figure 4



Use figures 1, 2, 3, and 4 to answer the following questions.

10. **a.** What happens to the date when travellers cross the date line going from east to west?

b. What happens to the date when travellers cross the date line going from west to east?

Check your answers by turning to the Appendix.

Did You Know?

When Fleming proposed the system of standard time, he realized that a meridian must be named to start the date for the world. If this was not done, there would be a disagreement about which meridian should be used (a new day begins in each time zone at midnight).

Fleming chose the meridian which runs along 180° for the date line. He chose this meridian because it is mostly over water and cuts through the smallest amount of land.

Today the International Date Line runs mostly along this meridian. However it turns eastward through the Bering Strait to avoid dividing Siberia and then turns westward to include the Aleutian Islands with Alaska. South of the equator, it turns eastward again to allow certain island groups to have the same day as New Zealand.

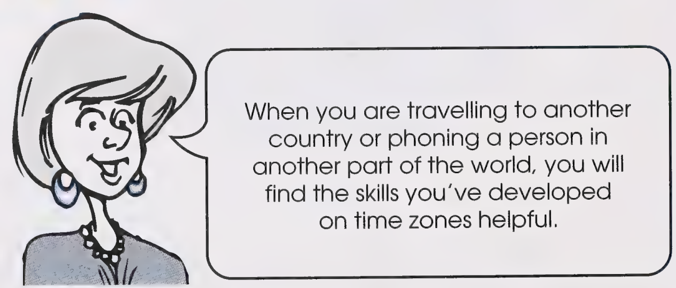
Now Try This

Use a problem-solving strategy to answer the following question.

11. The first four pentagonal numbers are 1, 5, 12, and 22.

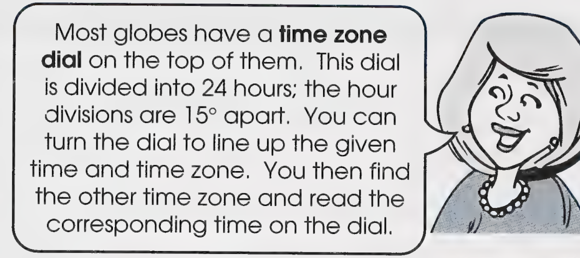
What are the next two pentagonal numbers?

Check your answers by turning to the Appendix.



Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.



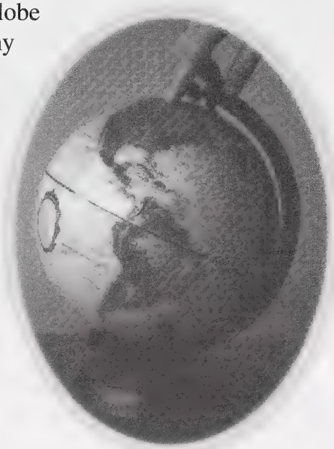
Extra Help

In this section you reviewed estimating and measuring length, mass, capacity, and angles. You found the importance of using standard measures. You calculated the time in different parts of the world.

If you had difficulty calculating the time in different parts of the world, you may find using a globe helpful.

Obtain a globe. (You will find a globe in most schools and libraries. Many homes also have globes.)

Find the prime meridian on the globe. Find the International Date Line on the globe. Notice their positions in relation to each other. Find the area where you live. Near which meridian of longitude do you live? In which time zone do you live? What is the time difference from your community to the prime meridian?



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The time zone dial on a globe may use the 24-hour clock or the A.M. and P.M. system like the one in the diagram.

Use a globe to answer question 1.

1. Find the time in each of the following places when it is 05:00 in Ottawa.

   a. Edmonton
   b. Halifax
   c. Paris, France
   d. Rome, Italy
   e. Tokyo, Japan
   f. Wellington, New Zealand

Check your answers by turning to the Appendix.

Another source of help with time zones is your telephone directory. To assist people with international telephone calls, many telephone companies include a time-difference chart in their directories.

A chart like the following is helpful for parts of Alberta, British Columbia, Saskatchewan, and the Northwest Territories (places that use Mountain Standard Time).

International Calling		
Alberta Locations to	**Country Code**	**Time Difference from Mountain Standard Time**
Argentina	54	+4
Brazil	55	+3
Britain	44	+7
China	86	+15
Denmark	45	+8
Egypt	20	+9
Ethiopia	251	+11
France	33	+8
Greece	30	+9
Israel	972	+9
Japan	81	+16
Malaysia	60	+15
Nigeria	234	+9
South Africa	27	+9
Thailand	66	+14

Note: The chart entitled "International Calling" does not make allowances for more than one time zone in a country. For example, Brazil has two time zones; only one time difference is given.

Example 1

What time and day is it in Israel when it is 07:00 on Monday in Alberta?

Solution

Step 1: Read the time difference from the "International Calling" chart.

The time difference is +9 h.

Step 2: Find the time and day in Israel.

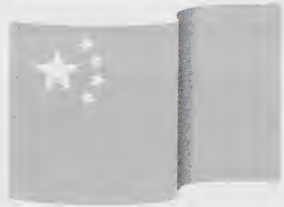
$$\begin{array}{r} 07:00 \\ +\quad 9 \\ \hline 16:00 \end{array}$$

Regrouping is **not** required. So, it is the same day.

When it is 07:00 on Monday in Alberta, it is 16:00 on Monday in Israel.

Note: You can think of this calculation as turning the clock **ahead** 9 h (from 07:00 to 16:00).

Example 2



What time and day is it in China when it is 18:00 on Friday in Alberta?

Solution

Step 1: Read the time difference from the "International Calling" chart.

The time difference is $+15$ h.

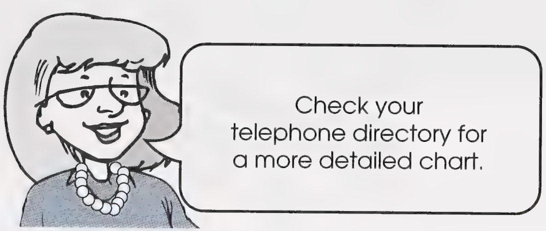
Step 2: Find the time and day in China.

$$\begin{array}{r} 18:00 \\ +\ 15 \\ \hline 33:00 = 09:00 \end{array}$$

Regrouping is required; 33:00 on Friday is the same as 09:00 on Saturday.

When it is 18:00 on Friday in Alberta, it is 09:00 on Saturday in China.

Note: You can think of this calculation as turning the clock **ahead** 15 h (from 18:00 to 09:00). You turn the clock ahead past midnight, so it is the next day.



Use the chart entitled "International Calling" to answer question 2.

2. Calculate the time and day in each of the following places when it is 12:00 on Wednesday in Alberta.

a. Argentina
b. Brazil
c. Egypt
d. South Africa
e. Malaysia
f. Thailand

Check your answers by turning to the Appendix.

Enrichment

When you calculate time in the Northern Hemisphere, you may also have to take into consideration the **daylight-saving time** system.

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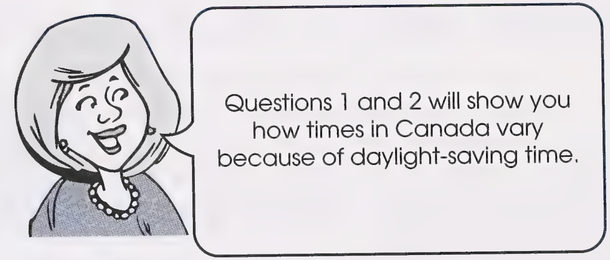
Daylight-saving time is a system for turning clocks ahead one hour in the spring to extend the daylight hours during the time when most people are awake. Clocks are returned to standard time in the fall by turning the clocks back one hour.

To help people remember this time change, a saying has been coined.

However, daylight-saving time is not uniformly practised.

- In Canada and the United States, daylight-saving time begins on the first Sunday in April and ends on the last Sunday in October.
- In Britain and some other countries, daylight-saving time lasts from March 30 to October 26.
- In most of the countries of western Europe, daylight-saving time begins on the last Sunday in March and ends on the last Sunday in September.
- Not all countries, provinces, or states use daylight-saving time. For example, Saskatchewan uses only standard time.



1. During the standard time period, when it is 09:00 in Vancouver, what time is it in each of the following places?

a. Edmonton **b.** Regina **c.** Winnipeg **d.** Toronto

2. During the period of daylight-saving time, when it is 09:00 in Vancouver, what time is it in each of the following places?

a. Edmonton **b.** Regina **c.** Winnipeg **d.** Toronto

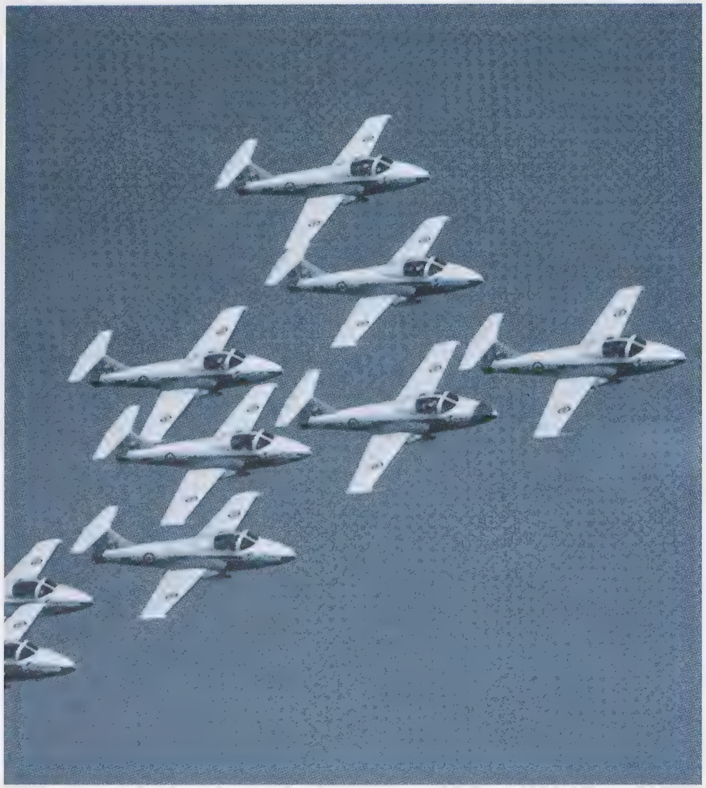
Check your answers by turning to the Appendix.

Now Try This

You may use the Internet to help you find the local time in different areas of the world. **Hint:** This is the uniform resource locator (URL) of a site that you may find helpful.

http://pathfinder.com/vibe/vibeworld/worldmap.html

Conclusion



Estimating and measuring are important skills in the everyday world. In this section you estimated and measured length, mass, capacity, and angles. You used scientific notation to describe large measurements. You calculated time and date changes that occur as you travel long distances and cross time zones. You used maps, globes, and many different kinds of measuring tools and instruments.

You have discovered that there are many factors to consider when estimating and measuring. Think of all the planes that travel around the world every day. What types of measurements must be considered when scheduling air flights?

Assignment

Assignment Booklet

You are now ready to complete the module assignment for Section 1.

Section 2: Motion Geometry

Do you ski? The skier in this photograph seems to be enjoying the powdered snow conditions.

Skiers generally make turns as they make their way down the hill. Sometimes skiers ski straight down a portion of the hill, especially when racing. Ski jumpers may even do flips. These ski movements can be described by a branch of mathematics called motion geometry.

In this section you will explore motion geometry. You will investigate slides, flips, and turns. You will also discover how slides, flips, and turns make some designs symmetrical.

Activity 1: Slides

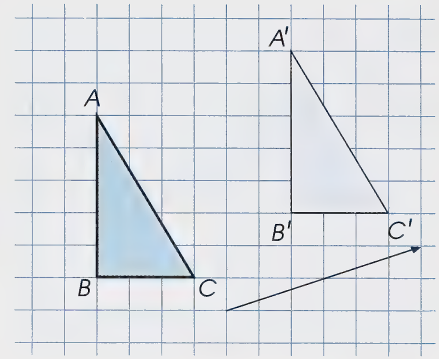
As a young child, did you enjoy using a slide on the playground?

In geometry, a slide (also called a **translation**) is a motion in which a figure is moved in a straight line from its original position to a new position.

The new position of the figure after the slide is called the slide image.

The slide arrow indicates the direction and distance the figure is moved or translated.

Example 1



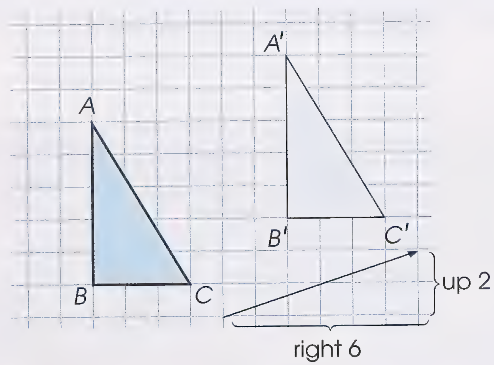
Example 1 shows the original position of a triangle and the new position (the slide image).

The vertices of the original triangle are labelled A, B, and C. The vertices of the slide image are labelled A', B', and C'.

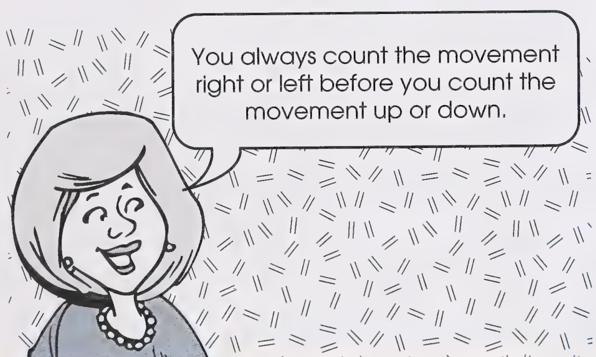
Read the accent mark that is above and to the right of each letter as "prime."

Notice that the matching vertices A and A', B and B', C and C' are each in a straight line.

The slide arrow in Example 1 indicates the direction and distance of the slide.

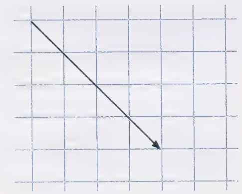


To get to the top of the slide arrow from the bottom of the slide arrow, you must count right 6 and up 2. Therefore, the triangle is moved in a straight line according to the rule (Right 6, Up 2) or (R6, U2).

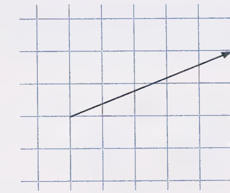


1. Give the rule for each of the following slide arrows.

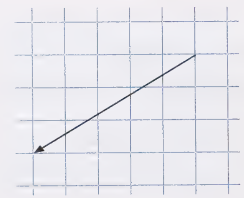
a.



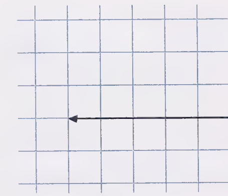
b.



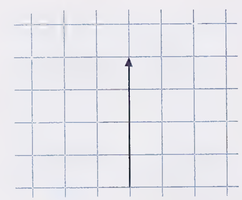
c.



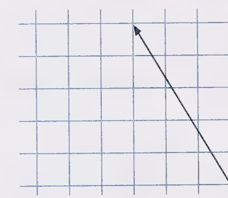
d.

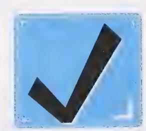


e.



f.



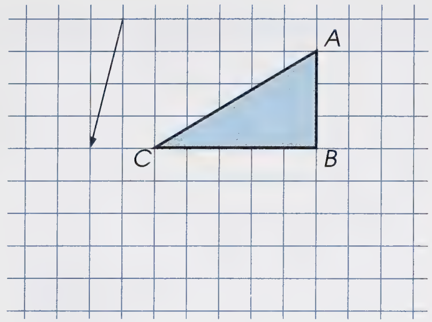


Check your answers by turning to the Appendix.

The following example shows how to draw the slide image of a figure.

Example 2

Draw the slide image for the given slide arrow.

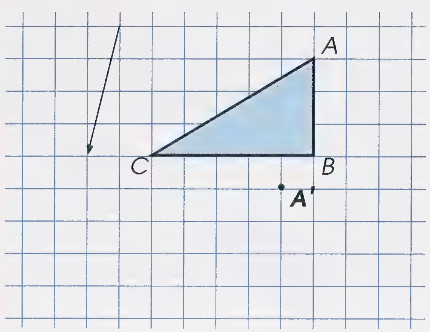


Solution

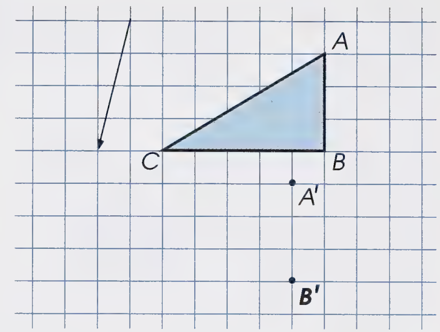
Step 1: Give the rule for the slide arrow.

The slide rule is (Left 1, Down 4) or (L1, D4).

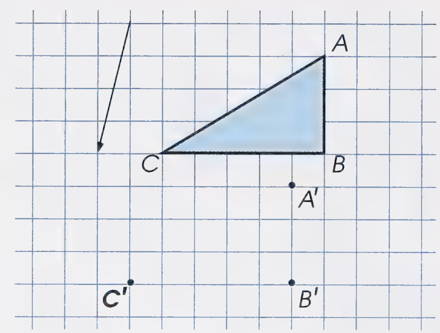
Step 2: Put your pencil tip on A. Count left 1 and down 4. Put a dot at that position and label it A'.



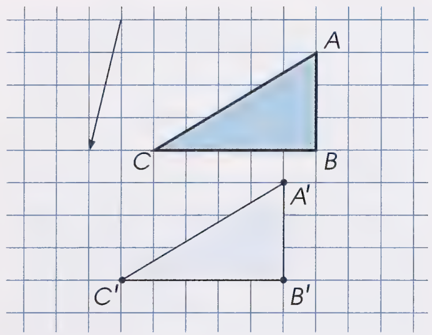
Step 3: Put your pencil on B. Count left 1 and down 4. Put a dot on that position and label it B'.



Step 4: Put your pencil on C. Count left 1 and down 4. Put a dot on that position and label it C'.



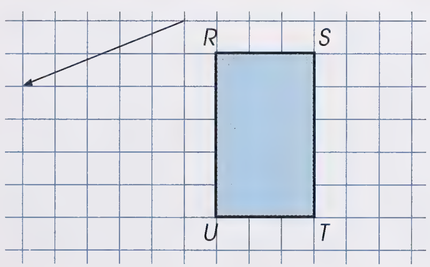
Step 5: Join the dots to show the slide image.



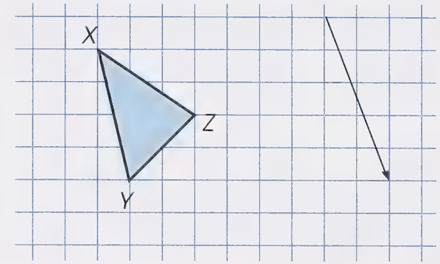
$\Delta A'B'C'$ is the slide image of ΔABC.

2. Copy each of the following figures onto graph paper. Then draw and label the slide image for the given slide arrow.

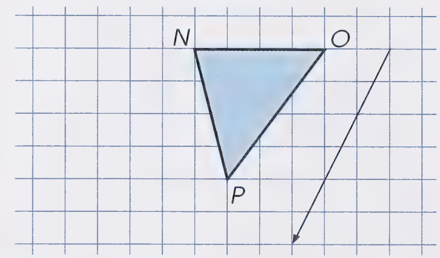
a.



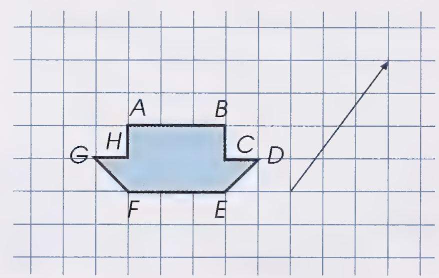
b.



c.

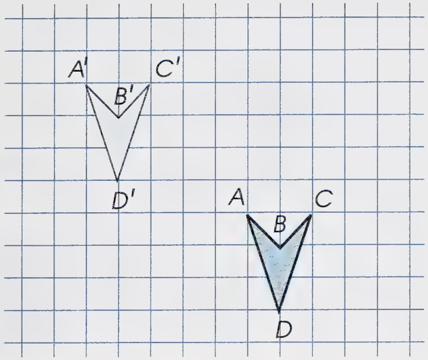


d.

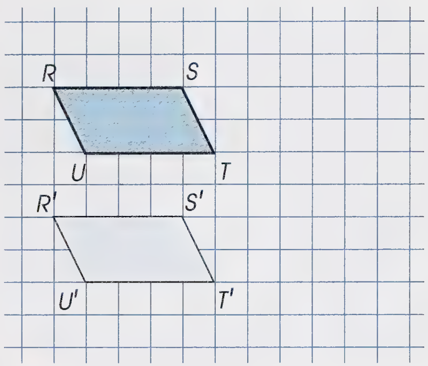


3. Give the slide rule for each of the following slides.

a.



b.



Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following questions.

4. The number □ 42 73 □ is divisible by 72. What is the first digit of the number? What is the last digit of the number?

5. In a series of three races, a runner earns 5 points for first place, 3 points for second place, and 1 point for third place; no ties are allowed. What is the smallest number of points a runner must earn in the three races to be guaranteed winning more points than any other runner?

Check your answers by turning to the Appendix.

Activity 2: Flips

Do you enjoy flipping the pages of a book or magazine?

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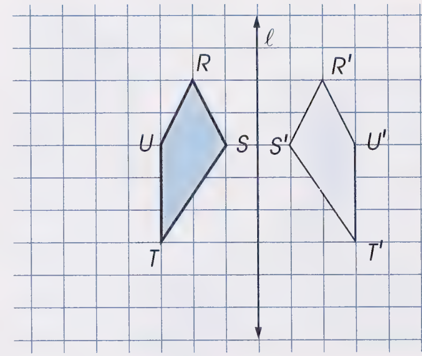
In geometry, a flip (also called a **reflection**) is a motion in which a figure is flipped or reflected over a line.

The new position of the figure after the flip is called the flip image or **reflection**.

The line over which the figure is flipped is called the flip line.

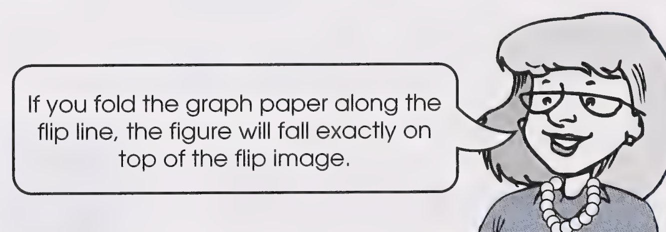
Example 1



Example 1 shows the original position of a four-sided figure (a quadrilateral) and its new position (the flip image).

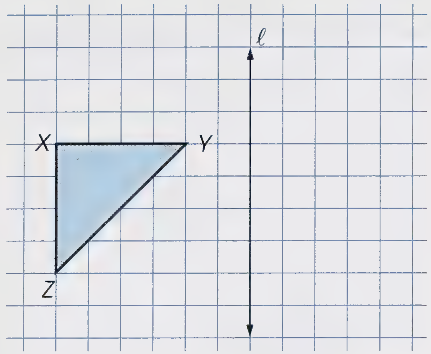
The vertices of the original quadrilateral are labelled R, S, T, and U. The vertices of the slide image are labelled R', S', T', and U'.

Notice that the flip line is labelled ℓ. The figure and its flip image are on opposite sides of the flip line. The matching vertices are the same distance from the flip line.



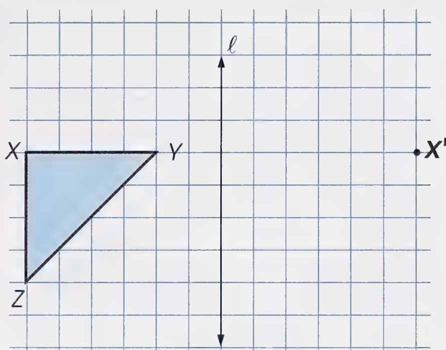
Example 2

Draw the flip image for the given flip line.

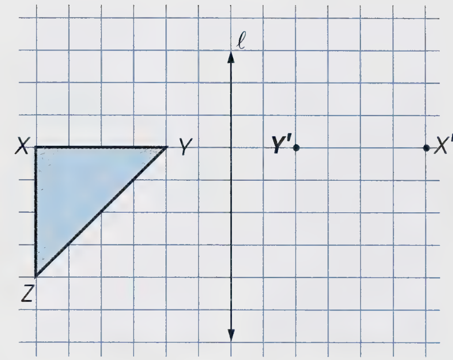


Solution

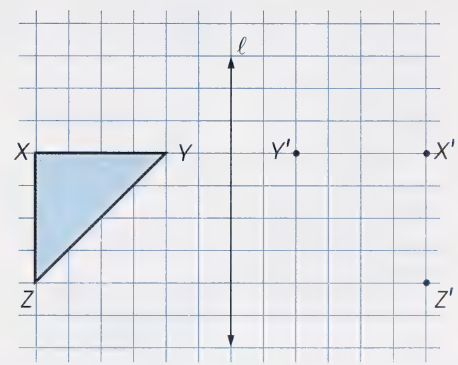
Step 1: Put your pencil on X. Count the number of units that X is from the flip line. Count the same number of units on the other side of the flip line and make a dot. Label it X'.



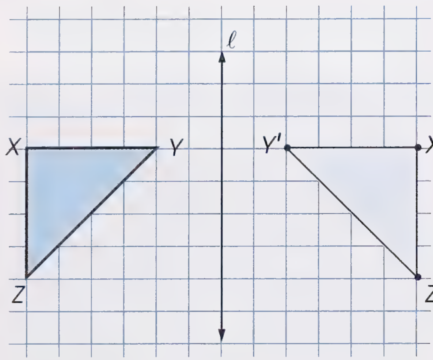
Step 2: Put your pencil on Y. Count the number of units that Y is from the flip line. Count the same number of units on the other side of the flip line and make a dot. Label it Y'.



Step 3: Put your pencil on Z. Count the number of units that Z is from the flip line. Count the same number of units on the other side of the flip line and make a dot. Label it Z'.



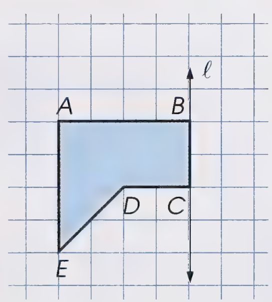
Step 4: Connect the dots to show the flip image.



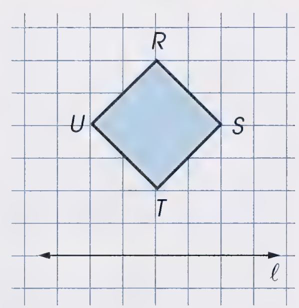
$\Delta X'Y'Z'$ is the flip image of ΔXYZ.

1. Copy each of the following figures onto graph paper. Then draw and label the flip image for the given flip line.

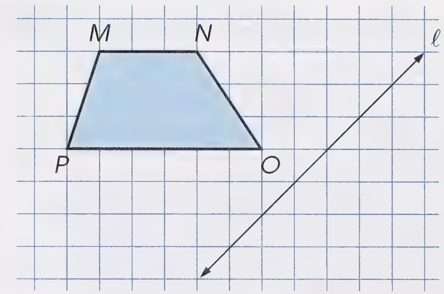
a.



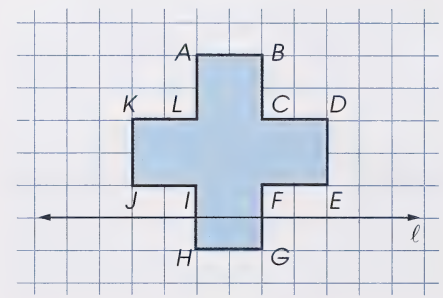
b.



c.

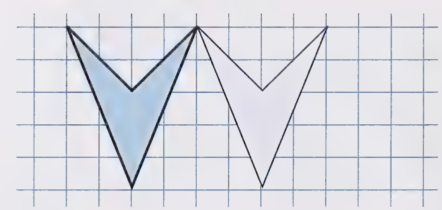


d.

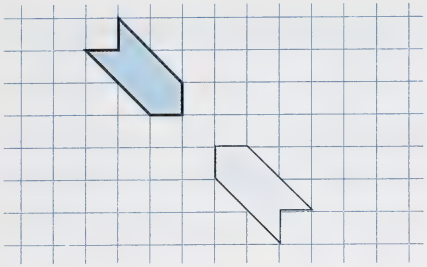


2. Copy each of the following figures and flip images on graph paper. Then show the flip line.

a.



b.

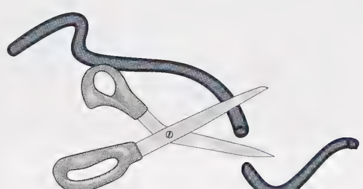


Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following questions.

3. A piece of string is 62 cm long. Explain how you could cut the string into two pieces so that one piece is three times the length of the other piece. You may **not** use a ruler.



4. There were 120 people at a picnic. In total, 75 people ate hamburgers. Of these, 26 ate both hamburgers and hot dogs. If 16 people did not eat either hamburgers or hot dogs, what was the total number of people who ate hot dogs?

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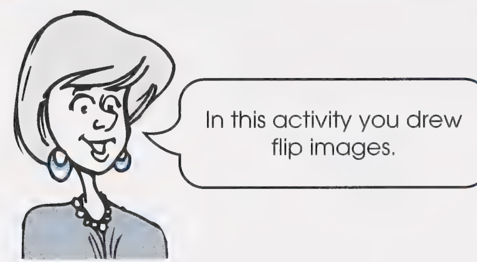
Check your answers by turning to the Appendix.



Activity 3: Turns

Do you enjoy riding on a Ferris wheel as it turns?

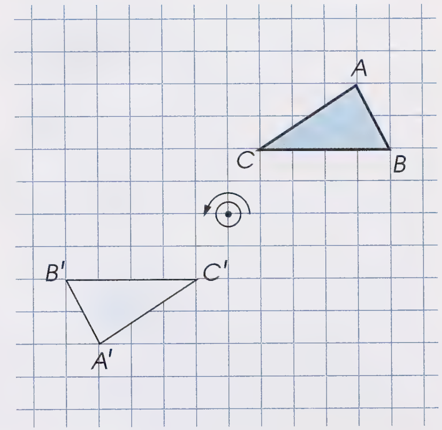
In geometry, a **turn** (also called a **rotation**) is a motion in which a figure is turned or rotated around a fixed point.

The new position of the figure is called the **turn image**.

A **turn centre** indicates the point about which the figure is turned. For emphasis, a circle is often placed around the turn centre.

A **turn arrow** indicates the direction and angle of the turn.

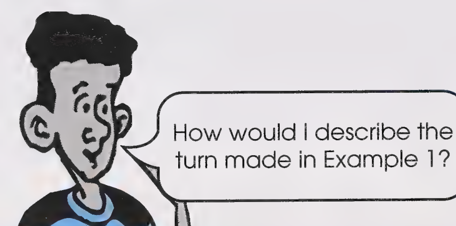
Example 1



Example 1 shows the original position of a triangle and its new position (the turn image).

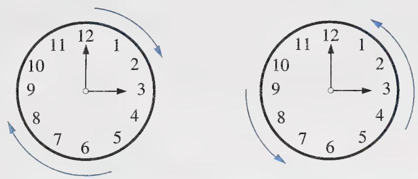
The vertices of the original triangle are labelled A, B, and C. The vertices of the turn image are labelled A', B', and C'.

A turn is described by angle and direction.



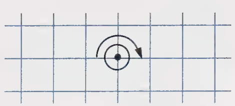
The angle of a turn can be described in two ways—as a fraction of a turn or in degrees.

The direction of a turn can be described as clockwise (cw) or counterclockwise (ccw). Remember, the hands on a clock move in a clockwise direction. The opposite direction is called counterclockwise.

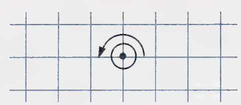


clockwise   counterclockwise

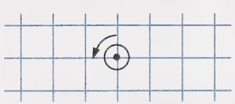
Here are some examples of turn arrows.



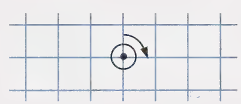
$\frac{1}{2}$ turn cw or 180° turn cw



$\frac{1}{2}$ turn ccw or 180° turn ccw

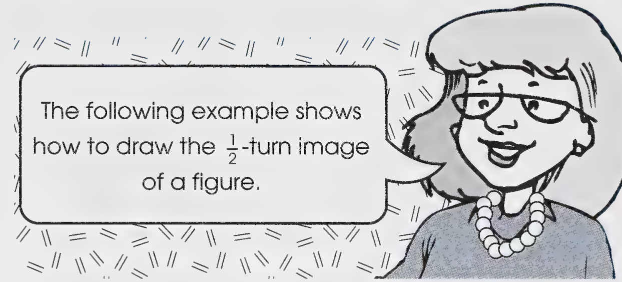


$\frac{1}{4}$ turn ccw or 90° turn ccw



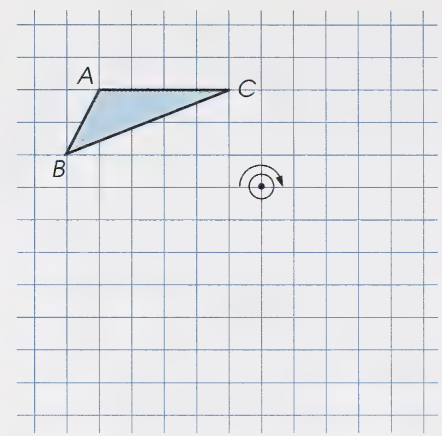
$\frac{1}{4}$ turn cw or 90° turn cw

In Example 1, the turn arrow indicates that ΔABC was rotated $\frac{1}{2}$ turn ccw.



Example 2

Draw and label the $\frac{1}{2}$-turn image for the given turn centre.

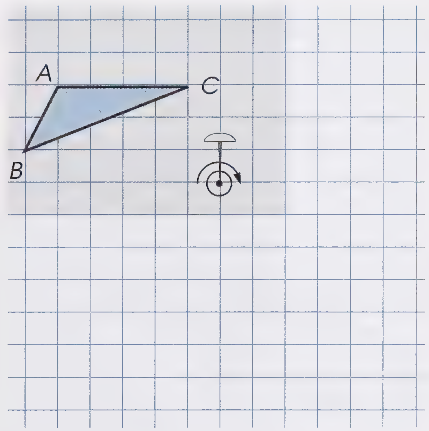


Solution

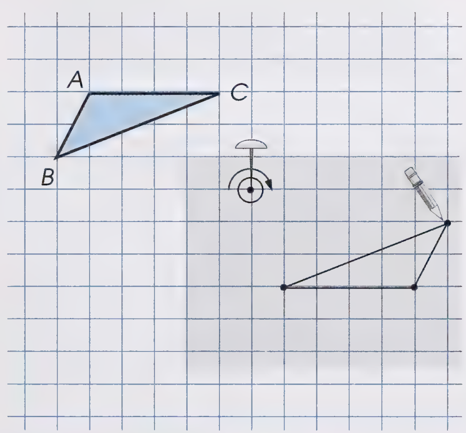
Step 1: Note the direction of the turn.

The turn is clockwise.

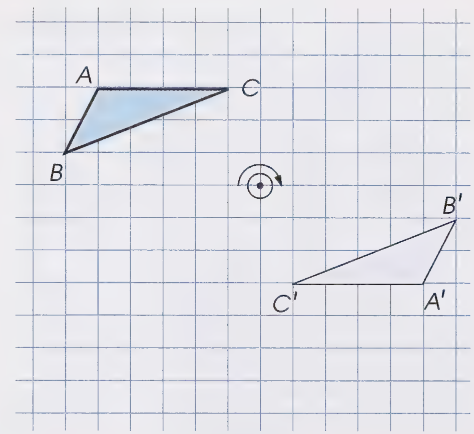
Step 2: Cover the figure and turn centre with tracing paper. Push a pin through the tracing paper into the turn centre. Then trace the figure.



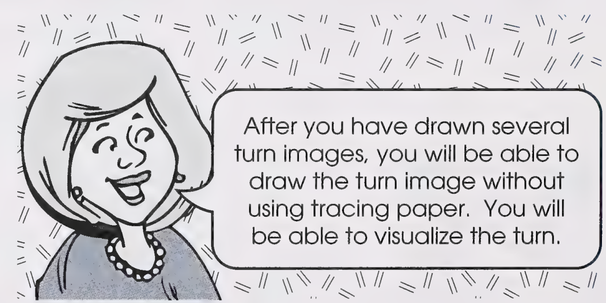
Step 3: Turn the tracing paper 180° clockwise around the turn centre. Mark the vertices of the figure on the tracing paper by pressing hard with a pencil.



Step 4: Remove the tracing paper and pin. Draw the turn image by connecting the impressions of the vertices that were marked. Label the vertices of the turn image.

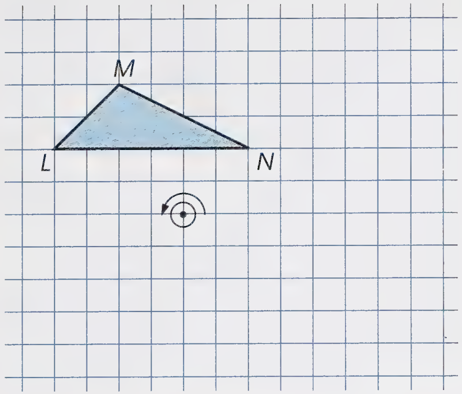


So, in Example 2, ΔABC was rotated $\frac{1}{2}$ turn clockwise.

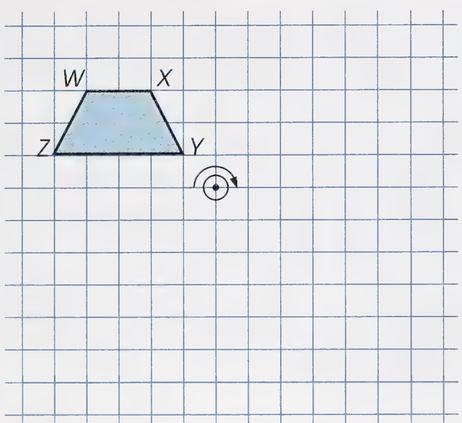


1. Copy each of the following figures onto graph paper. Then draw and label the turn image for the given turn centre and arrow.

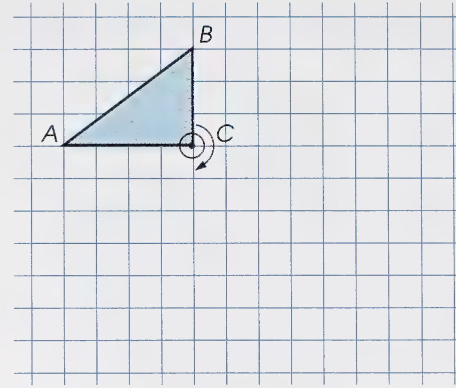
a.



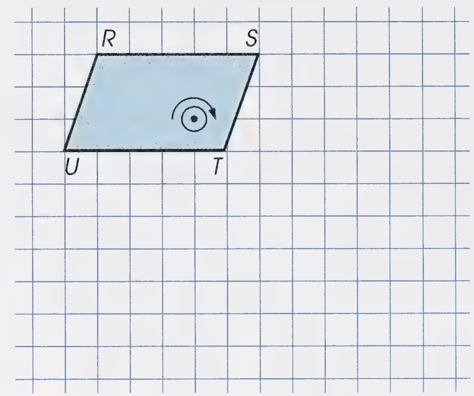
b.



c.



d.

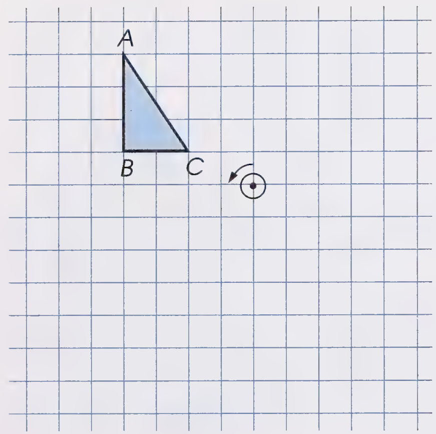


Check your answers by turning to the Appendix.

The following example shows how to draw the $\frac{1}{4}$-turn image of a figure.

Example 3

Draw and label the $\frac{1}{4}$-turn image for the given turn centre and arrow.

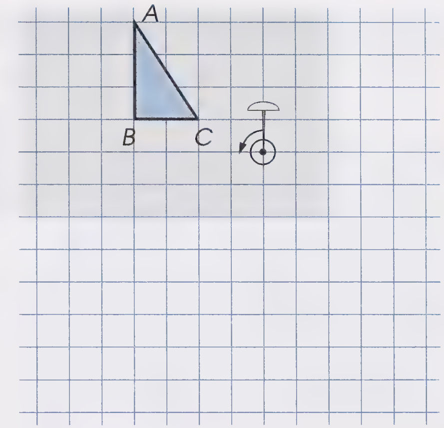


Solution

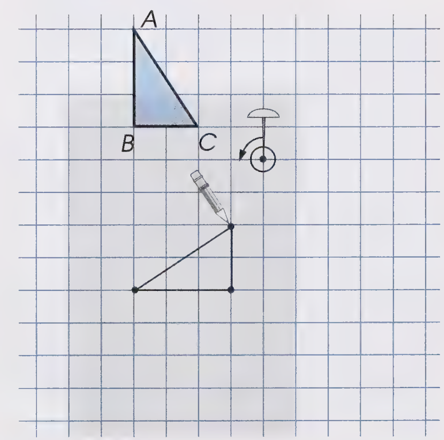
Step 1: Note the direction of the turn.

The turn is counterclockwise.

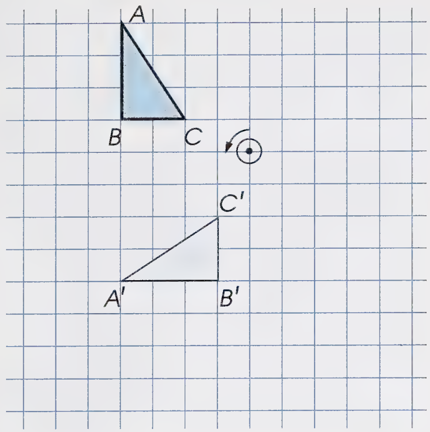
Step 2: Cover the figure and turn centre with tracing paper. Push a pin through the tracing paper into the turn centre. Then trace the figure.



Step 3: Turn the tracing paper 90° counterclockwise around the turn centre. Mark the vertices of the figure on the tracing paper by pressing hard with a pencil.



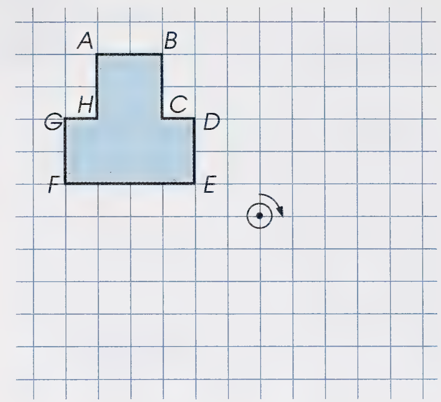
Step 4: Remove the tracing paper and pin. Draw the turn image by connecting the impressions of the vertices that were marked. Label the vertices of the image.



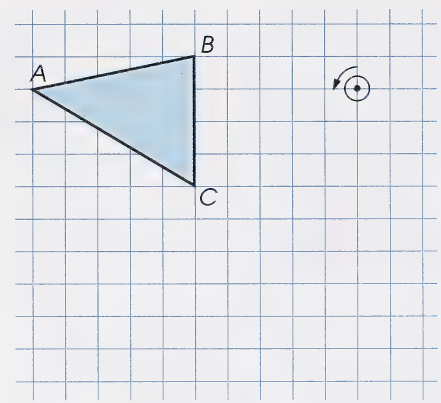
So, in Example 3, ΔABC was rotated $\frac{1}{4}$ turn counterclockwise.

2. Copy each of the following figures onto graph paper. Then draw and label the turn image of each figure for the given turn centre and arrow.

a.

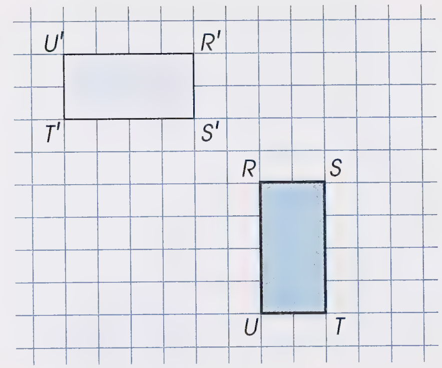


b.

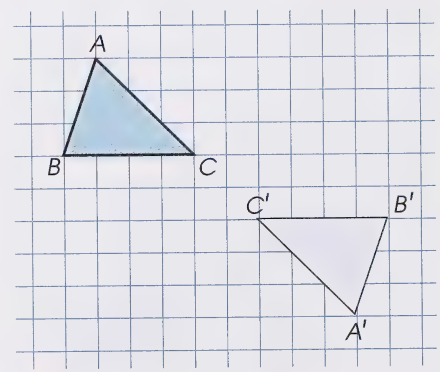


3. Copy the following figures and their turn images onto graph paper. Indicate the turn centre and turn arrow for each figure.

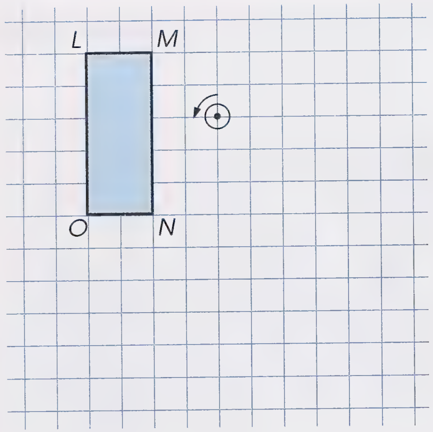
a.



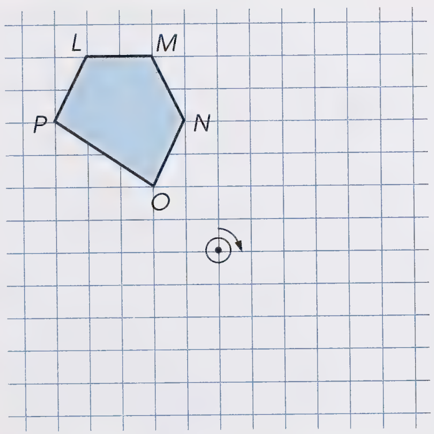
b.



c.



d.



Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

4. Copy and complete the following chart.

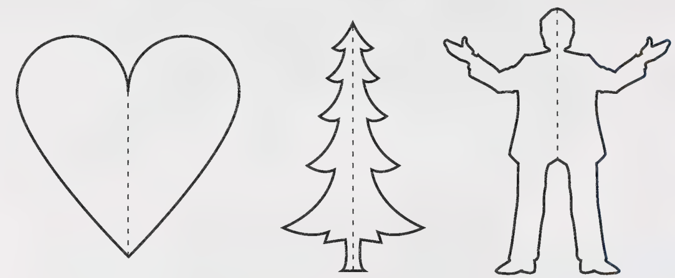
1	4	9	16			
4	9	16				
9	16					
16						

Check your answer by turning to the Appendix.

Activity 4: Symmetry

Flip Symmetry

Have you ever made designs like these by folding and cutting paper?



Each of these designs has flip symmetry.

Flip symmetry is the property whereby one-half of a figure can be flipped onto the other half; it is the condition whereby one-half of a figure is the mirror image of the other half. Flip symmetry may also be called **line symmetry** or **reflection symmetry**.

The flip line which divides a figure into two congruent parts is called the line of symmetry. Congruent parts are exactly the same size and shape.

If a figure has flip symmetry, the line of symmetry goes through the centre of the figure. Every part of the figure on one side of the line is balanced by a **corresponding part** or matching part on the other side.

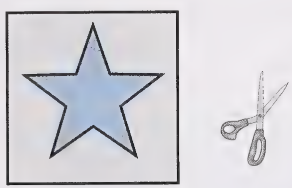
You can test for flip symmetry using tracing paper.

Example

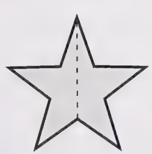
Does this star have flip symmetry? If so, how many lines of symmetry does it have?

Solution

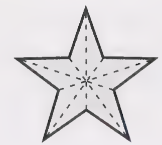
Step 1: Trace the figure and cut out the tracing.



Step 2: Test to see if the star has flip symmetry. Can you fold the cutting so that the left side matches the right side?



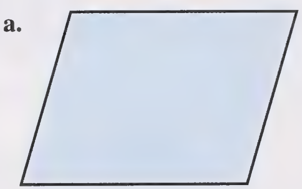
Step 3: Test to see how many different ways the cutting can be folded so that the left side matches the right side.



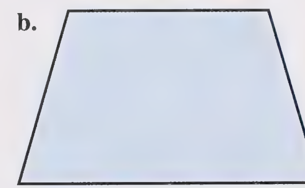
Yes, the figure has flip symmetry; there are five lines of symmetry.

1. Use tracing paper to test each of the following figures for flip symmetry. Does each figure have flip symmetry? Answer **yes** or **no**. If yes, state how many lines of symmetry each figure has.

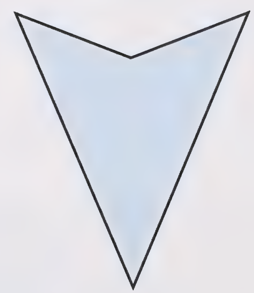
a.



b.



c.



d.

e. 

f. 

Check your answers by turning to the Appendix.

There are many examples of objects with flip symmetry in the everyday world.

- The building in the following photograph has flip symmetry.

- A butterfly has flip symmetry.

- A person's face has flip symmetry.

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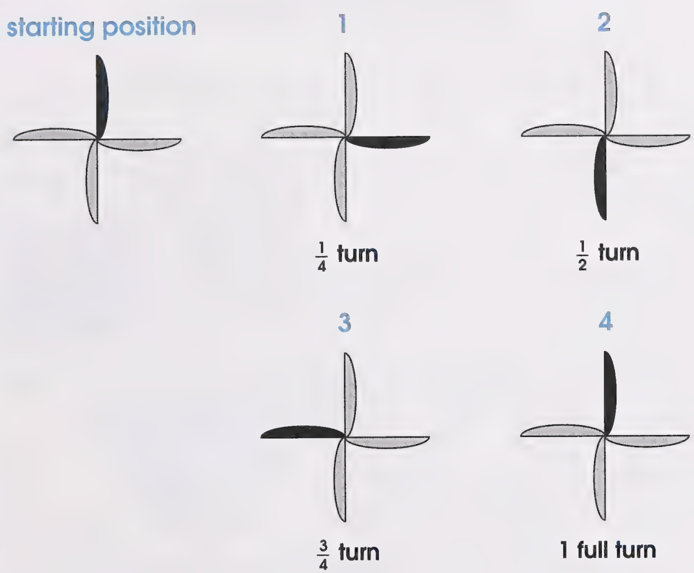
2. Find photographs in magazines or newspapers of objects with flip symmetry.

Share the photographs with your learning facilitator.

Turn Symmetry

When you were younger did you ever play with a pinwheel? (A pinwheel has blades similar to that of a fan, a windmill, or a propeller. When a child holds up the pinwheel, the wind makes it whirl.)

The pinwheel does not have flip symmetry; it cannot be divided into two congruent parts with a line. However, the pinwheel can be turned around its centre in such a way that it is in the same position more than once in a full turn.

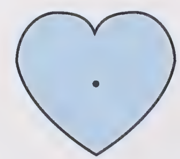


The pinwheel has **turn symmetry**. In one full turn, the pinwheel is in its original position 4 times; its **order of turn symmetry** is 4.

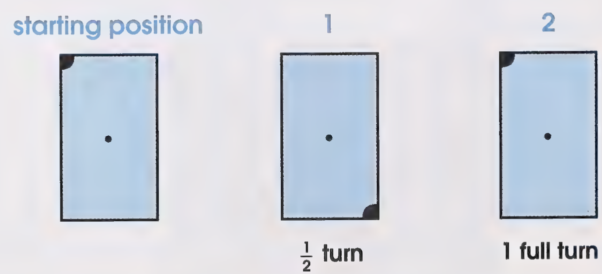
Turn symmetry is the property that a figure coincides with its original position more than once in a full turn. Turn symmetry may also be called **rotational symmetry** or **point symmetry**.

The turn centre about which a figure with turn symmetry may be rotated is called the **point of symmetry**.

The following figure does not have turn symmetry. In one full turn, the heart is in exactly the original position only once.



The following figure has turn symmetry. In one full turn, the rectangle is in the original position more than once; its order of turn symmetry is 2.



You can use tracing paper and a pin to test for turn symmetry.

Example

Does this star have turn symmetry? If yes, what is the turn order?

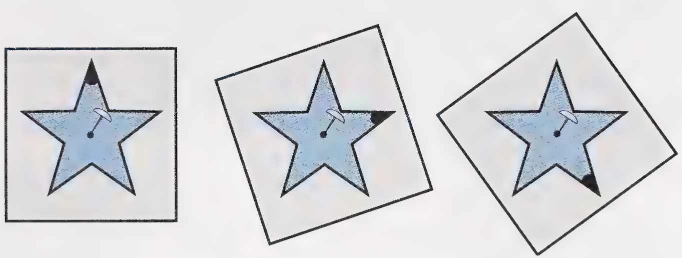
Solution

Step 1: Trace the figure. To keep track of the positions as the star turns, mark one point on the tracing of the star.

Put a pin through the turn centre.

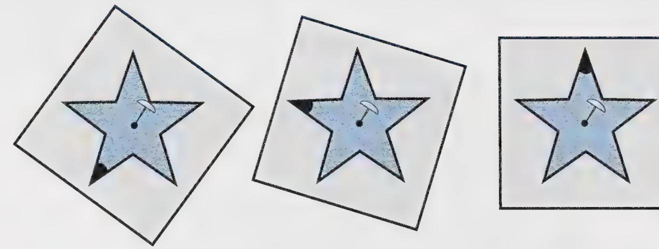
Step 2: Test to see if the star has turn symmetry. Does the figure coincide with its original position more than once in a full turn?

starting position | 1 | 2



$\frac{1}{5}$ turn | $\frac{2}{5}$ turn

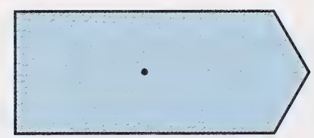
3 | 4 | 5

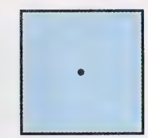


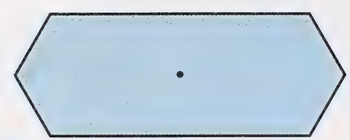
$\frac{3}{5}$ turn | $\frac{4}{5}$ turn | 1 full turn

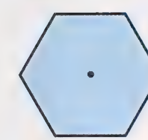
The star has turn symmetry. In one full turn, the star is in the original position more than once; its order of turn symmetry is 5.

3. Use tracing paper to test each of the following figures for flip symmetry. Does each figure have turn symmetry? Answer **yes** or **no**. If yes, state the turn order.

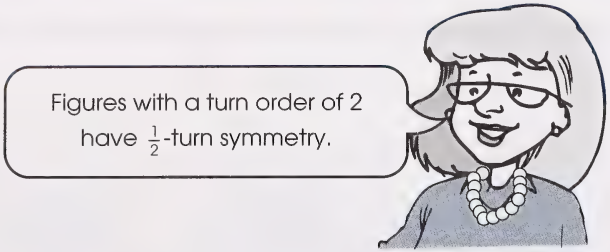
a. 

b. 

c. 

d. 

Check your answers by turning to the Appendix.



4. Does each of the following letters have $\frac{1}{2}$-turn symmetry? Answer **yes** or **no**.

a. A b. N c. F

d. E e. M f. Z

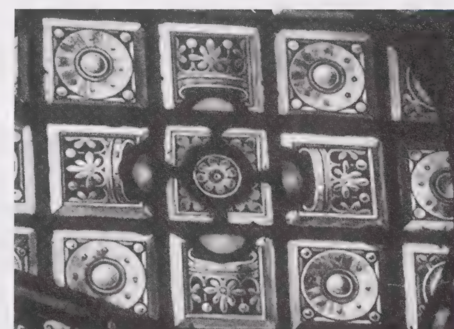
g. K h. H i. X

Check your answers by turning to the Appendix.

There are many interesting examples of turn symmetry in the everyday world.

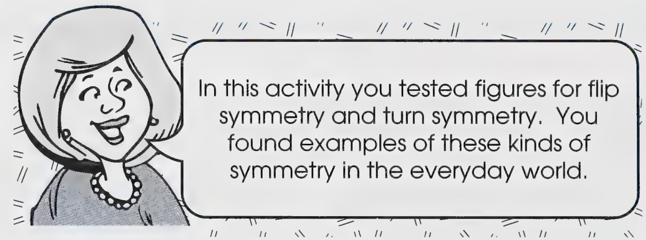
- The flower in this photograph has turn symmetry.

- The stained-glass window in this photograph has turn symmetry.



5. Find photographs of objects with turn symmetry in magazines or newspapers.

Share the photographs with your learning facilitator.



Now Try This

Use a problem-solving strategy to answer the following question.

6.

What is the number that has the following characteristics?

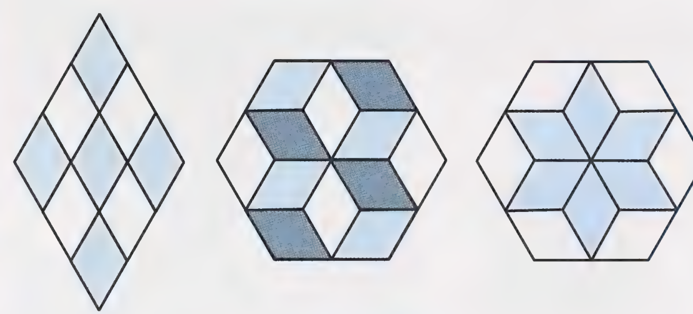
- It is less than 80.
- If you divide it by 2, the remainder is 1.
- If you divide it by 3, the remainder is 2.
- If you divide it by 4, the remainder is 3.
- If you divide it by 5, the remainder is 4.
- If you divide it by 6, the remainder is 5.

Check your answer by turning to the Appendix.

Activity 5: Applications of Slides, Flips, and Turns

Many designs are created by sliding, flipping, or turning identical shapes in a tiling pattern.

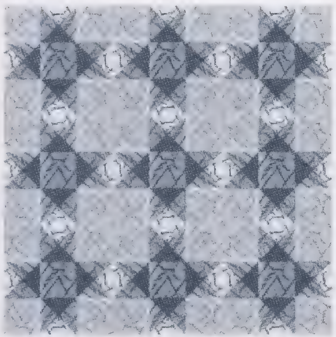
The following designs are made with identical-shaped bricks of different colours.

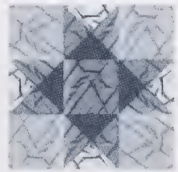


Brick Designs

Quilt Designs

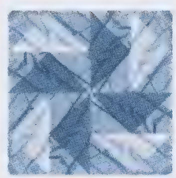
The following quilt is made of identical squares. The pattern is called the "Variable Star." Notice that each square of the quilt has flip symmetry and turn symmetry.





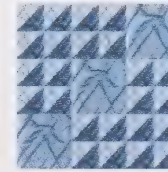
One square of the quilt.

The following quilt is made of identical squares. The pattern is called the "Dutchman's Puzzle." Notice that each square has turn symmetry.



One square of the quilt.

This quilt is made of identical squares. The pattern is called the "Cut Glass Dish." Notice that each square of the quilt does **not** have flip symmetry or turn symmetry.



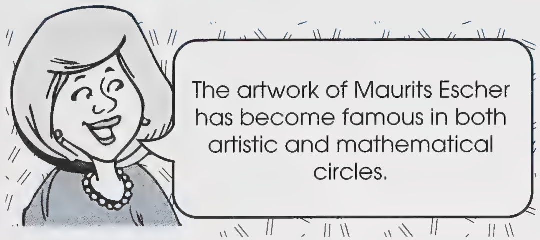
One square of the quilt.

The brick and quilt designs are tessellations.

A tessellation is an arrangement of congruent figures that covers a surface without gaps or overlapping.

View the video *Math Vantage: Tessellations* to discover more about tessellations.

The Dutch artist Maurits Escher (1898–1972) used tessellations in his artwork.



Each of the following designs was made by Escher.

- This design, called "Horsemen," is a tessellation made by flipping and sliding a figure over an area.

- This design, called "Lizards," is a tessellation made by sliding and turning a figure over an area.

Use the Internet to view other Escher designs. This is the uniform resource locator (URL) of an interesting site.

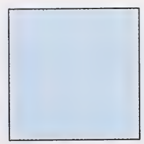
http://www.texas.net/escher/gallery/gallerym.html

You can make Escher-like designs using a **template**.

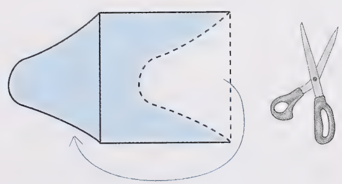
A template is a stiff piece of material used as a pattern.

Example 1

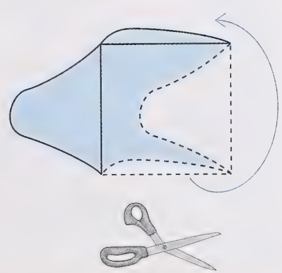
Step 1: Start with a figure you know will tessellate and make a template on stiff paper. (This example uses a square.)



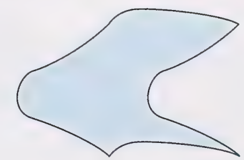
Step 2: Cut a shape out of the template. Slide the shape to the opposite side of the template and tape it securely.



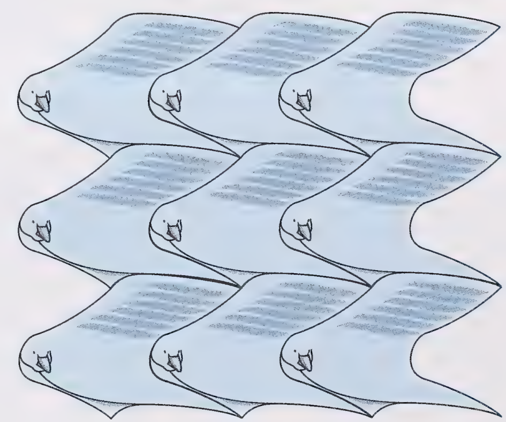
Step 3: Cut a shape out of another side of the template. Slide the shape to the opposite side of the template and tape it securely.



Step 4: Try to imagine what your template represents. (Perhaps you think the shape looks like a bird.)



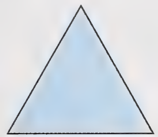
Step 5: Position the template on a sheet of paper and cover the page by tessellating the shape. Afterwards, colour the design to illustrate what you imagined the shape to be.



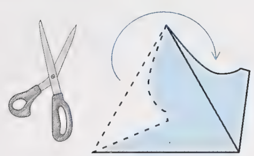
Note: This design, called "Wild Birds," was created by sliding the template.

Example 2

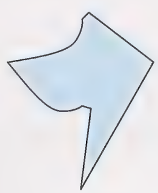
Step 1: Start with a shape you know will tessellate and make a template on stiff paper. (This example uses an equilateral triangle.)



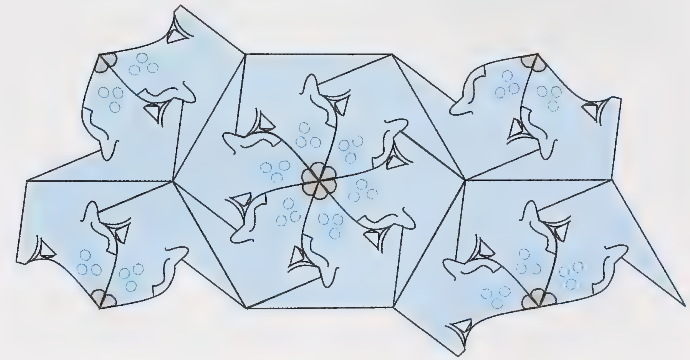
Step 2: Cut a shape out of the template. Rotate the shape to another side and tape it securely.



Step 3: Try to imagine what your template looks like. (Does this template look like a dog's head?)



Step 4: Position the template on a sheet of paper and cover the page by tessellating the shape. Afterwards, colour the design to illustrate what you imagined the shape to represent.



Note: This design, called "Dogs," was created by turning and sliding the template.

1. Make an Escher-like design of your own.

If you have access to a computer and the program *TesselMania!*™, you may enjoy doing question 2.

2. Use the program *TesselMania!*™ to create an Escher-like design on the computer.

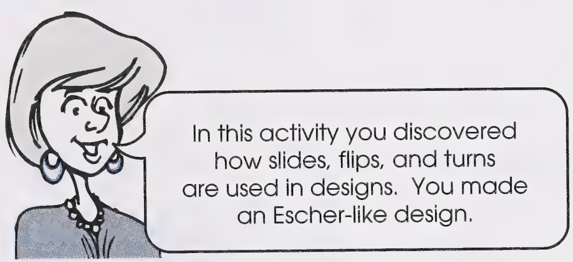
Share your design(s) with your learning facilitator.

Now Try This

Use a problem-solving strategy to answer the following question.

3. How many times in a day do the hands on this clock point in the same direction?

Check your answers by turning to the Appendix.



Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

In this section you explored flip symmetry.

A **mira** can be used to test for flip symmetry.

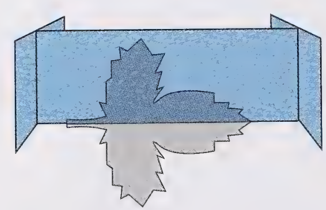
A mira is an instrument made of semitransparent plastic.

Example

Does this leaf shape have flip symmetry?

Solution

Place the mira on the figure. If the mira reflects one-half of the leaf shape exactly on the other half, the leaf shape has flip symmetry.



The mira is on the line of symmetry.

The mira reflects one-half of the leaf exactly on the other half. The leaf shape has flip symmetry.

Use a mira to answer questions 1, 2, and 3.

1. Does each of the following letters have flip symmetry? Answer **yes** or **no**. If yes, how many lines of symmetry are there?

a. A **b.** N **c.** F **d.** E

e. M **f.** Z **g.** K **h.** H

2. Does each of the following aboriginal designs have flip symmetry? Answer yes or no. If yes, how many lines of symmetry are there?

a.

Northwest

b.

Southwest

c.

Micmac

d.

Plains

e.

Inuit

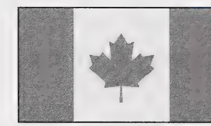
f.

Ecuador

3. Does each of the following flags have flip symmetry? Answer **yes** or **no**. If yes, how many lines of symmetry are there?

a.

Nigeria

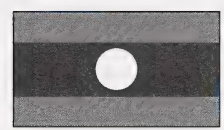
b.

Canada

c.

Cuba

d.

United States of America

e.

Laos

f.

United Kingdom

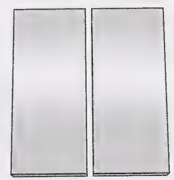
Check your answers by turning to the Appendix.

Enrichment

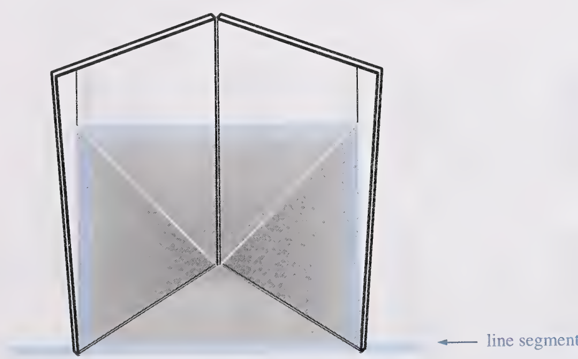
Kaleidoscopes

Did you ever hold a kaleidoscope up to the light and turn the tube? A kaleidoscope uses a pair of hinged mirrors to produce several reflections.

1. Make a simple kaleidoscope by using two identical purse-sized rectangular mirrors. Place the two mirrors **face down** as shown in the diagram. The distance between the mirrors should be about twice their thickness. Join the mirrors with a strip of masking tape.



Next, with a coloured pencil, draw a line segment on a blank page of paper. Position the hinged mirrors on the line segment so that the line segment is reflected in each mirror.



a. Adjust the angle of the hinged mirrors until you see a triangle in the mirrors. Estimate the measurement of the angle formed by the hinged mirrors.

b. Adjust the angle of the hinged mirrors until you see a square in the mirrors. Estimate the measurement of the angle formed by the hinged mirrors.

c. Adjust the angle of the hinged mirrors until you see a pentagon in the mirrors. Estimate the measurement of the angle formed by the hinged mirrors.

d. What happens to the size of the angle formed by the hinged mirrors as the number of sides of the figure created increases?

2. Place the hinged mirrors on the dotted lines in the given diagram and view the design created in your kaleidoscope. Draw a sketch of the design you see in your kaleidoscope.

Check your answers by turning to the Appendix.

The kaleidoscope was invented by Sir David Brewster in 1816. Discover more interesting facts about the kaleidoscope by visiting a website such as the following:

http://www.eiu.edu/ac/busi/lum/kr.html

Conclusion

In this section you explored motion geometry. You made slide, flip, and turn images of figures. You tested figures for flip symmetry and turn symmetry. You created Escher-like designs by sliding, flipping, and/or turning a template that you had made.

The next time you ski or watch someone skiing, notice the skier's motions. Does the skier slide, flip, and turn?

Assignment

Assignment Booklet

You are now ready to complete the module assignment for Section 2.

Section 3: Angles and Lines

Have you ever looked at a building that is being constructed and noticed the different angles that are formed by the intersecting lines? Have you ever thought about how geometry is used in the everyday world?

Architects and engineers must think about angles and lines when they are planning and constructing buildings, bridges, and highways. Angles and lines are important in the design and function of these projects.

In this section you will classify angles and lines. You will also draw angles and lines using different geometric tools and instruments.

Activity 1: Classifying Angles and Lines

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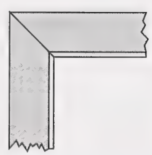
Knowledge about angles and lines is important in many jobs in the everyday world. For example, carpenters need to know about angles and lines when they mitre corners of door frames or join baseboards.



mitred corner of door frame

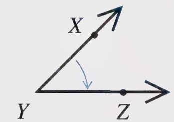
joined baseboard

In this activity you will classify individual angles, pairs of angles, and pairs of lines. You will also use your knowledge of angles and lines to solve problems.

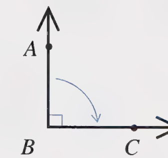
Classifying Angles by Size

Individual angles can be classified according to their size.

- If the size of an angle is less than a quarter turn, the angle is called an **acute angle**. For example, $\angle XYZ$ is an acute angle. It measures 45°.

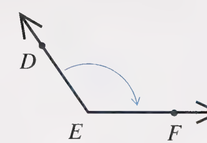


- If the size of an angle is exactly a quarter turn, the angle is called a **right angle**. For example, $\angle ABC$ is a right angle. It measures 90°.

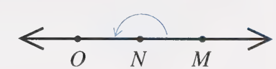


The symbol ⊾ indicates a right angle.

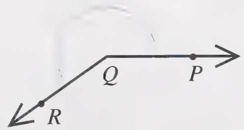
- If the size of an angle is more than a quarter turn and less than a half turn, it is called an **obtuse angle**. For example, $\angle DEF$ is an obtuse angle. It measures 125°.



- If the size of an angle is exactly a half turn, the angle is called a **straight angle**. For example, $\angle MNO$ is a straight angle. It measures 180°.

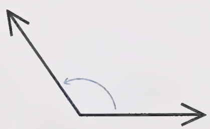


- If the size of an angle is more than half a turn and less than a full turn, it is called a **reflex angle**. For example, $\angle PQR$ is a reflex angle. It measures 215°.

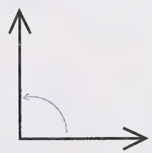


1. Without measuring, tell whether each of the following is a right angle, an acute angle, an obtuse angle, a straight angle, or a reflex angle.

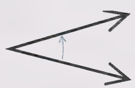
a.



b.

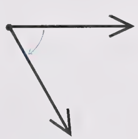


c.

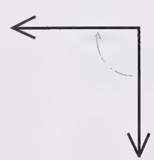


d.

e.



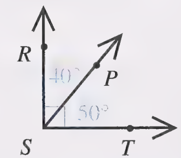
f.



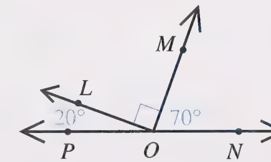
Check your answers by turning to the Appendix.

Angles can be classified by their sum.

- Angles that have a sum of 90° are **complementary angles**. For example, $\angle RSP$ and $\angle PST$ are complementary angles; $\angle LOP$ and $\angle MON$ are also complementary angles.

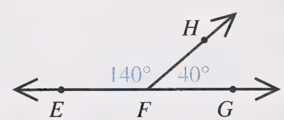


$40° + 50° = 90°$

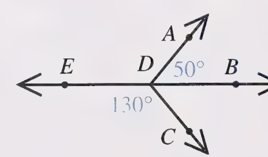


$20° + 70° = 90°$

- Angles that have a sum of 180° are **supplementary angles**. For example, $\angle EFH$ and $\angle HFG$ are supplementary angles; $\angle ADB$ and $\angle EDC$ are also supplementary angles.



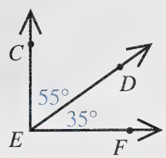
$140° + 40° = 180°$



$50° + 130° = 180°$

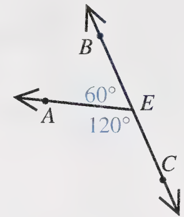
2. Use the given diagram to complete each of the following statements.

a.



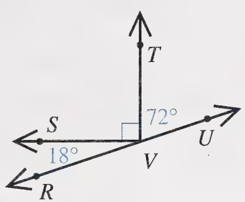
$\angle CED$ and $\angle DEF$ are ________ angles.

b.



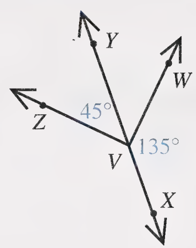
$\angle AEB$ and $\angle AEC$ are ________ angles.

c.



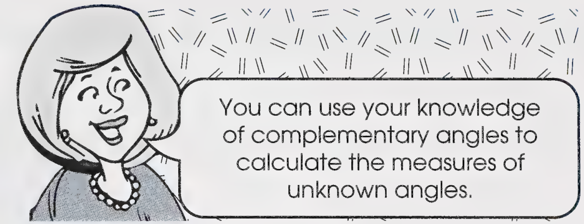
$\angle RVS$ and $\angle TVU$ are ________ angles.

d.



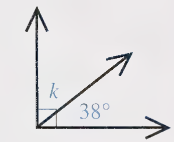
$\angle YVZ$ and $\angle WVX$ are ________ angles.

Check your answers by turning to the Appendix.



Example 1

Calculate the measure of the unknown angle. Do **not** use a protractor.

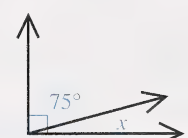


Solution

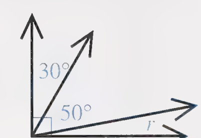
Statement	Reason
$k + 38° = 90°$	The sum of complementary angles is 90°.
$\therefore\ k = 52°$	

3. Calculate the measure of each of the unknown angles. Do **not** use a protractor.

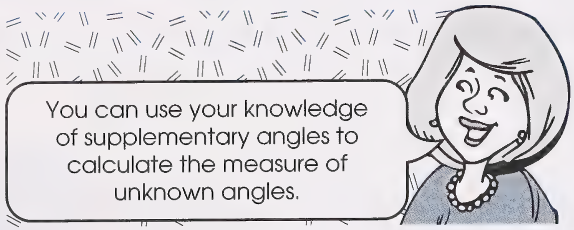
a.



b.



Check your answers by turning to the Appendix.



Example 2

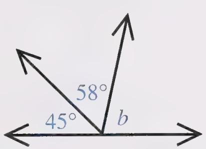
Calculate the measure of the missing angle.
Do **not** use a protractor.

Solution

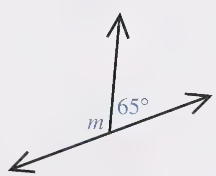
Statement	Reason
$n + 150° = 180°$	The sum of supplementary angles is 180°.
$\therefore\ n = 30°$	

4. Calculate the measure of each of the unknown angles. Do **not** use a protractor.

a.



b.



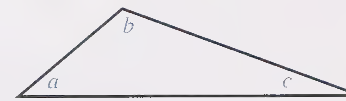
Check your answers by turning to the Appendix.

Angles are formed where the sides of a figure meet.

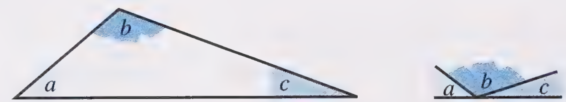
The angles inside the figure are called **interior angles**.

Example 3

This triangle has three interior angles. The angles are labelled a, b, and c.



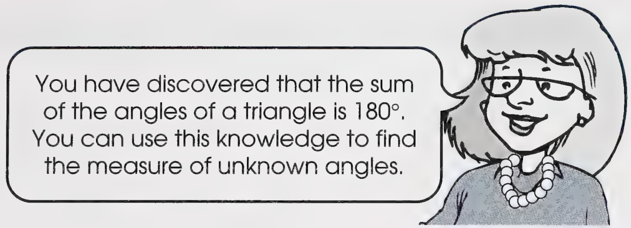
Note: If you cut out the triangle in the example and carefully tear off the three corners, the pieces can be placed side by side.



Because the angles form a straight angle, you can conclude that the sum of the angles of ΔABC is 180°.

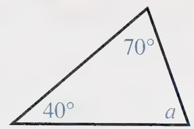
5. Draw several triangles that are of different sizes and shapes. Cut out each triangle and carefully tear off the three corners of each. Then place the pieces side by side. Do the three angles of each triangle form a straight angle?

Check your answers by turning to the Appendix.



Example 4

Find the measure of the missing angle.



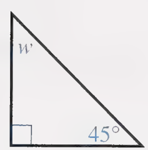
Solution

Statement	Reason
$70° + 40° + a = 180°$	The sum of the angles of a triangle is 180°.
$110° + a = 180°$	
$\therefore\ a = 70°$	

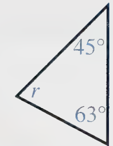
6. Find the measure of the missing angle in each of the following triangles. Do **not** use a protractor.

a.

b.



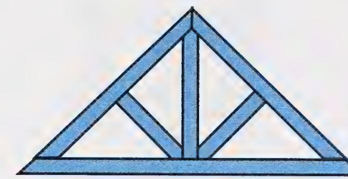
c.



d.

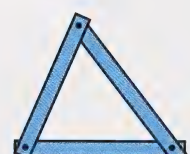
Check your answers by turning to the Appendix.

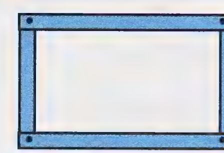
Did You Know?



Triangles are often used in the construction of buildings and bridges because triangles are **rigid**.

To demonstrate this, make a triangle and a quadrilateral (four-sided figure) with strips of paper and paper fasteners. Then try to change the angles in each figure.



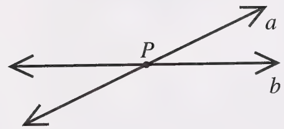


You will discover that the angles of a triangle are rigid; they cannot be changed. The angles in the quadrilateral, however, can be changed.

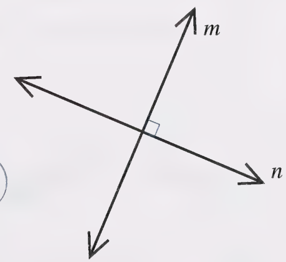
Classifying Lines

Pairs of lines can be classified according to their position.

- Lines that cross are called **intersecting lines**. For example, line a and line b are intersecting lines. The lines cross at point P.

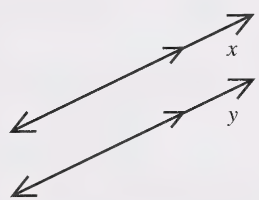


- Lines that cross at a right angle are called **perpendicular lines**. For example, line $m \perp$ line n.



The symbol $\perp$ is read as "is perpendicular to."

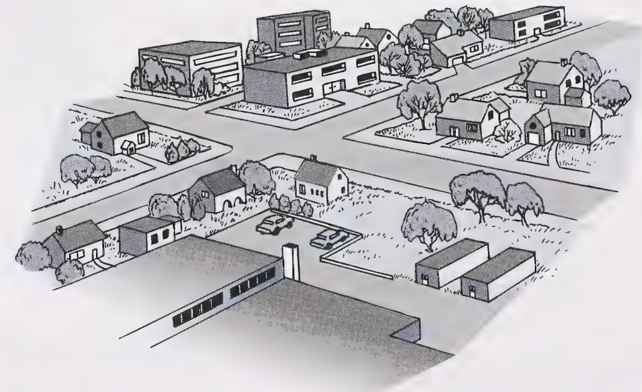
- Lines that stay the same distance apart and never cross are called **parallel lines**. For example, line $x \parallel$ line y.



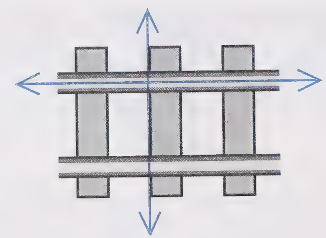
The symbol $\parallel$ is read as "is parallel to."

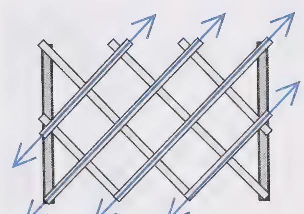
The → symbol on both lines indicates that the lines are parallel.

The pair of roads in this diagram suggest intersecting lines. The place where the roads meet is an **intersection**.



These railway tracks suggest perpendicular lines.





This baby gate suggests parallel lines.

7. Does each of the following letters contain parallel lines? Answer **yes** or **no**.

a. E **b.** V **c.** N

d. Z **e.** H **f.** X

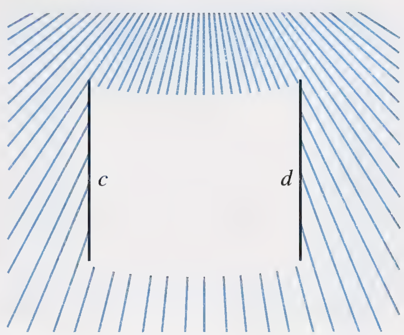
8. Does each of the following letters contain perpendicular lines? Answer **yes** or **no**.

a. E **b.** V **c.** N

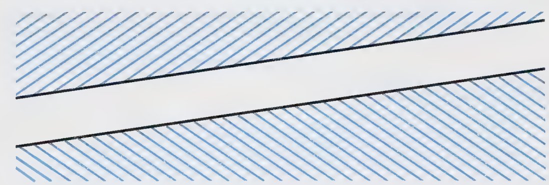
d. Z **e.** H **f.** X

9. Things are not always what they appear to be.

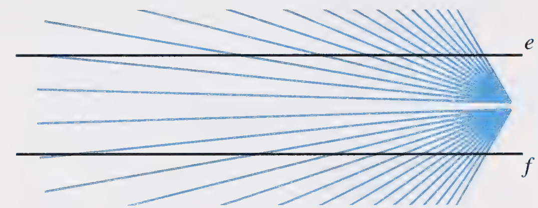
a. Are lines *c* and *d* parallel? Answer **yes** or **no**.



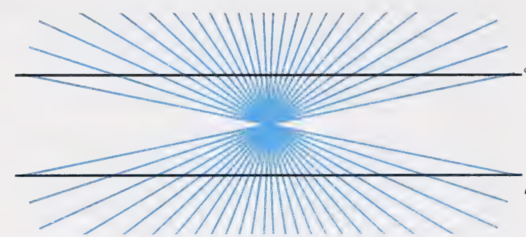
b. Are lines *a* and *b* parallel? Answer **yes** or **no**.



c. Are lines *e* and *f* parallel? Answer **yes** or **no**.



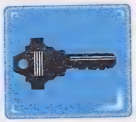
d. Are lines *g* and *h* parallel? Answer **yes** or **no**.



Check your answers by turning to the Appendix.

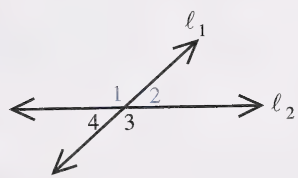
Classifying Angles Formed by Intersecting Lines

When two angles intersect, four angles are formed.



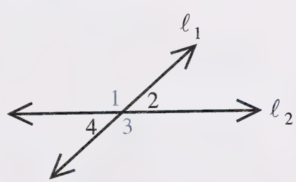
The pairs of angles formed by intersecting lines can be classified according to their position.

- Any two angles that are next to each other and share a common vertex and a common side are called adjacent angles. For example, $\angle 1$ and $\angle 2$ are adjacent angles. **Note:** $\angle 2$ and $\angle 3$, $\angle 3$ and $\angle 4$, and $\angle 1$ and $\angle 4$ are also adjacent angles.

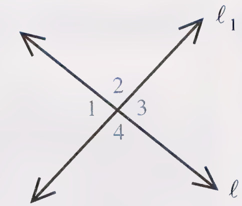


The word *adjacent* means having a common border. Farmland and other properties that are next to each other are called adjacent.

- Any two nonadjacent angles are called vertically opposite angles. For example, $\angle 1$ and $\angle 3$ are vertically opposite angles. **Note:** $\angle 2$ and $\angle 4$ are also vertically opposite angles. Vertically opposite angles may also be called **opposite angles** or **vertical angles**.

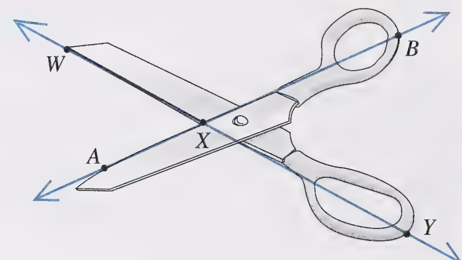


10. Complete each of the following statements using the information from the given diagram.



a. $\angle 2$ and $\angle 3$ are __________ angles.
b. $\angle 2$ and $\angle 4$ are __________ angles.
c. $\angle 3$ and $\angle 4$ are __________ angles.
d. $\angle 3$ and $\angle 1$ are __________ angles.

11. A pair of scissors suggests two intersecting lines. Use the diagram to complete each of the following statements.

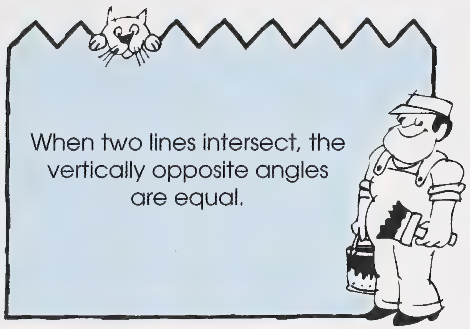


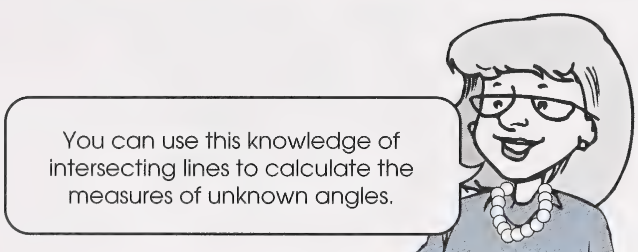
a. $\angle AXW$ and $\angle BXY$ are __________ angles.
b. $\angle WXB$ and $\angle BXY$ are __________ angles.
c. $\angle WXB$ and $\angle AXY$ are __________ angles.
d. $\angle WXA$ and $\angle AXY$ are __________ angles.

12. Draw several pairs of intersecting lines and measure the vertically opposite angles. What do you notice?

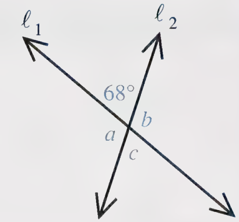
Check your answers by turning to the Appendix.

You have discovered another fact about angles and intersecting lines.





Example



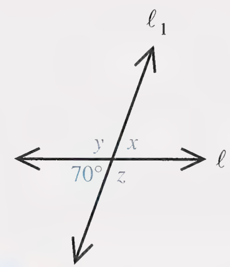
Calculate the measures of the unknown angles. Do **not** use a protractor.

Solution

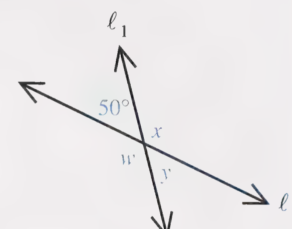
Statement	Reason
$c = 68°$	Vertically opposite angles are equal.
$a + 68° = 180°$	The sum of supplementary angles is 180°.
$\therefore\ a = 112°$	
$a = b$	Vertically opposite angles are equal.
$\therefore\ b = 112°$	

13. Calculate the measure of each of the unknown angles. Do **not** use a protractor.

a.

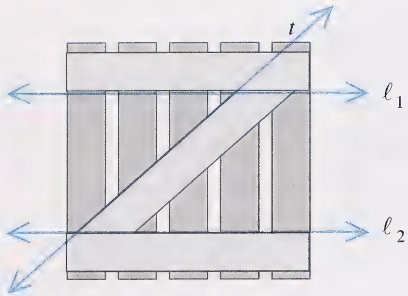


b.



Check your answers by turning to the Appendix.

Classifying Angles Formed by Parallel Lines and a Transversal

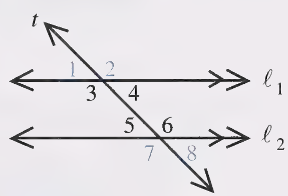


This fence suggests a pair of parallel lines cut by a transversal.

A transversal is a line that intersects two or more other lines.

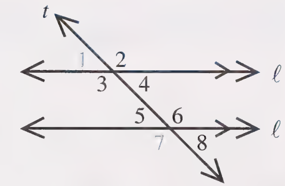
When a pair of parallel lines is cut by a transversal, eight angles are formed. These angles can be classified according to their position.

- Angles that are on the outside of the parallel lines are **exterior angles**. For example, $\angle 1$, $\angle 2$, $\angle 7$, and $\angle 8$ are exterior angles.



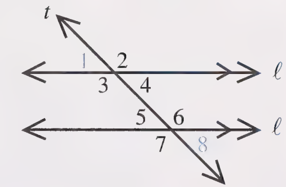
The → symbol on both lines indicates that the lines are parallel.

- Exterior angles on the same side of the transversal are called co-exterior angles. For example, $\angle 1$ and $\angle 7$ are co-exterior angles.



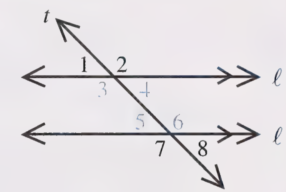
Note: $\angle 2$ and $\angle 8$ are also co-exterior angles.

- Exterior angles **not** on the same side of the transversal are called exterior alternate angles. For example, $\angle 1$ and $\angle 8$ are exterior alternate angles.

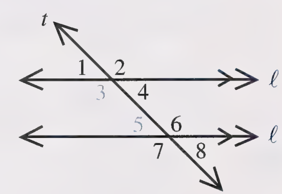


Note: $\angle 2$ and $\angle 7$ are also exterior alternate angles.

- Angles that are on the inside of the parallel lines are **interior angles**. For example, $\angle 3$, $\angle 4$, $\angle 5$, and $\angle 6$ are interior angles.

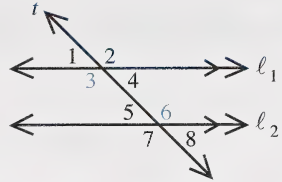


- Interior angles on the same side of the transversal are called co-interior angles. For example, $\angle 3$ and $\angle 5$ are co-interior angles.



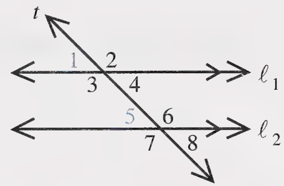
Note: $\angle 4$ and $\angle 6$ are also co-interior angles.

- Interior angles **not** on the same side of the transversal are called **interior alternate angles**. For example, $\angle 3$ and $\angle 6$ are interior alternate angles.



Note: $\angle 4$ and $\angle 5$ are also interior alternate angles.

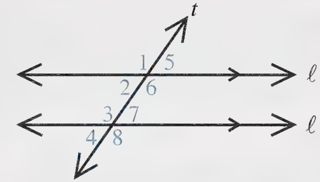
- If an angle can be slid along the transversal onto a second angle, then the pair are called **corresponding angles**. For example, $\angle 1$ and $\angle 5$ are corresponding angles.



Because corresponding angles can be matched through slides, corresponding angles are equal.

Note: $\angle 3$ and $\angle 7$, $\angle 2$ and $\angle 6$, and $\angle 4$ and $\angle 8$ are also corresponding angles.

14. Use the given diagram to complete the following statements.

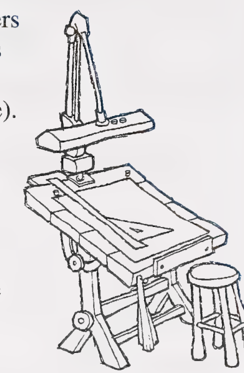


a. $\angle 5$ and $\angle 7$ are ______________ angles.
b. $\angle 5$ and $\angle 4$ are ______________ angles.
c. $\angle 5$ and $\angle 8$ are ______________ angles.
d. $\angle 2$ and $\angle 3$ are ______________ angles.
e. $\angle 2$ and $\angle 7$ are ______________ angles.
f. $\angle 2$ and $\angle 4$ are ______________ angles.

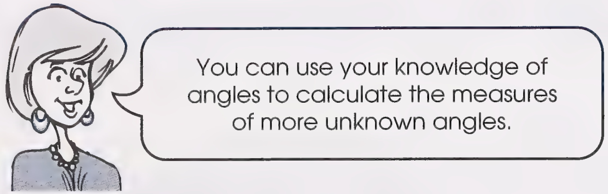
15. Graphic artists, architects, and engineers sometimes draw parallel line segments using two drafting tools—a **T square** and a **set square** (also called a triangle).

Research how these professionals use these tools to draw parallel line segments.

Note: A drafting table is shown in the given illustration. On the table are a T square and a set square (triangle).

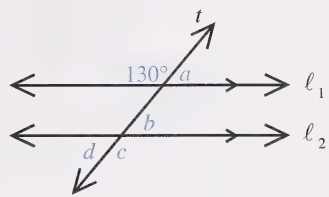


Check your answers by turning to the Appendix.



Example 1

Calculate the measures of the unknown angles. Do **not** use a protractor.



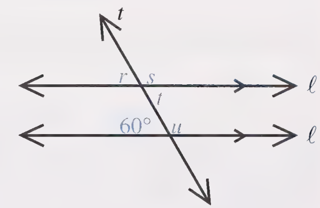
Solution

Statement	Reason
$\ell_1 \parallel \ell_2$	given
$a + 130° = 180°$	The sum of supplementary angles is 180°.
$\therefore\ a = 50°$	
$a = b$	Corresponding angles of parallel lines are equal.
$\therefore\ b = 50°$	
$b + c = 180°$	The sum of supplementary angles is 180°.
$50° + c = 180°$	
$\therefore\ c = 130°$	
$b = d$	Vertically opposite angles are equal.
$\therefore\ d = 50°$	

Note: In Example 1, $a = 50°$, $b = 50°$, $c = 130°$, and $d = 50°$; however, different statements and reasons may be given.

Example 2

Calculate the measures of the unknown angles. Do **not** use a protractor.



Solution

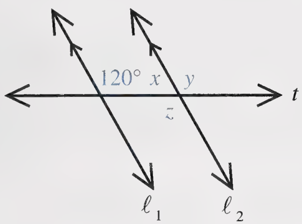
Statement	Reason
$\ell_1 \parallel \ell_2$	given
$r = 60°$	Corresponding angles of parallel lines are equal.
$r + s = 180°$	The sum of supplementary angles is 180°.
$60° + s = 180°$	
$\therefore\ s = 120°$	
$r = t$	Vertically opposite angles are equal.
$\therefore\ t = 60°$	
$s = u$	Corresponding angles of parallel lines are equal.
$\therefore\ u = 120°$	

Note: In Example 2, $r = 60°$, $s = 120°$, $t = 60°$, and $u = 120°$; however, different statements and reasons may be given.

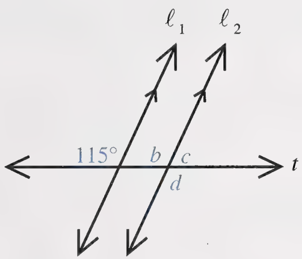
When you are answering questions about angles, you may find it helpful to make a sketch of the given diagram and to put the measures of the calculated angles on the sketch.

16. For each of the following diagrams, calculate the unknown angles. Do **not** use a protractor.

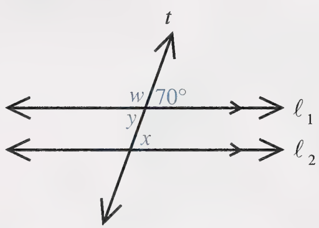
a.



b.



c.

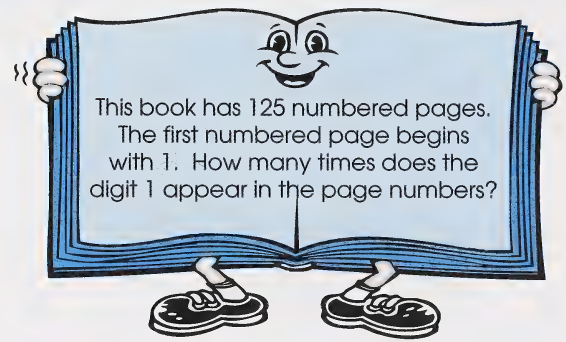


Check your answers by turning to the Appendix.

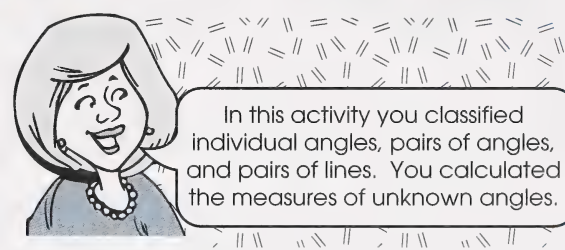
Now Try This

Use a problem-solving strategy to answer the following question.

17.



Check your answers by turning to the Appendix.



Activity 2: Drawing Angles

Have you ever considered being a graphic artist, an architect, or an engineer? These professionals use different mathematical instruments to draw angles.

In this activity you will draw angles using the tools in a mathematical instrument set.

Using a Protractor and Straightedge

You can draw any angle with a protractor and **straightedge**.

A straightedge is any object with at least one straight edge that can be used to draw **line segments** (a line segment is part of a line; it has two end points).

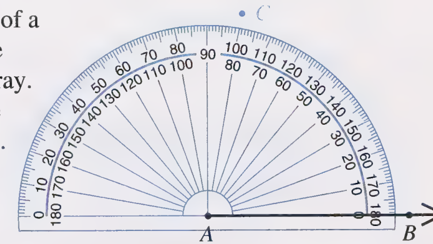
Example

Use a protractor and straightedge to draw an angle of 80°.

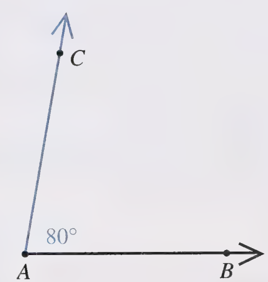
Solution

Step 1: Draw a ray AB with a straightedge. (A ray has only one end point.)

Step 2: Place the centre of a protractor on the endpoint of the ray. Line up the base line with the ray. Make a dot at 80° and label it C. (Make sure you use the correct scale.)



Step 3: Remove the protractor. With a straightedge, draw a second ray from the end point of the first ray through point C.



$\angle CAB = 80°$

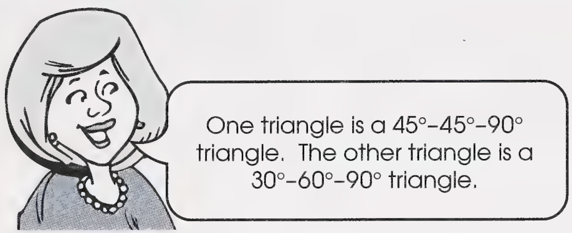
1. Use a protractor to draw angles with the following measures.

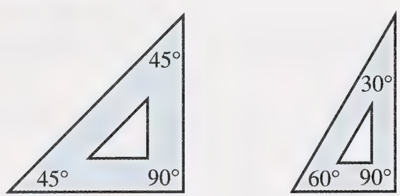
a. 90°	**b.** 45°	**c.** 135°
d. 60°	**e.** 30°	**f.** 120°

Check your answers by turning to the Appendix.

Using a Triangle and a Straightedge

In most mathematical instrument sets there are two plastic **triangles**.





You can draw **special angles** with a triangle and a straightedge.

Angles of 30°, 45°, 60°, and 90° are often referred to as special angles because they are used frequently in designs and carpentry projects.

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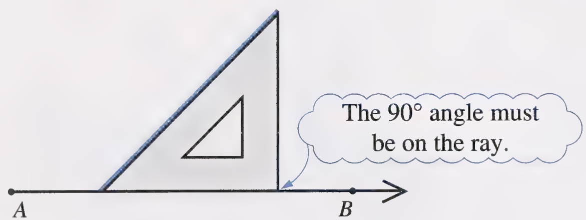
Example

Use a triangle and straightedge to construct an angle of 45°.

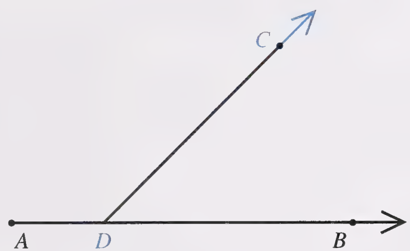
Solution

Step 1: Draw a ray AB with a straightedge.

Step 2: Place a 45°–45°–90° triangle on the ray and trace the edge of the triangle to create an angle of 45°.



Step 3: Remove the triangle and use a straightedge to complete the ray begun in Step 2. Label the ray *CD*.



$\angle CDB = 45°$

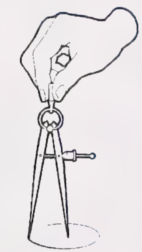
2. Use a straightedge and the 30°–60°–90° triangle in a mathematical instrument set to draw the following angles.

a. 30° **b.** 60° **c.** 90°

Check your answers by measuring the angles with a protractor.

Using a Compass and Straightedge

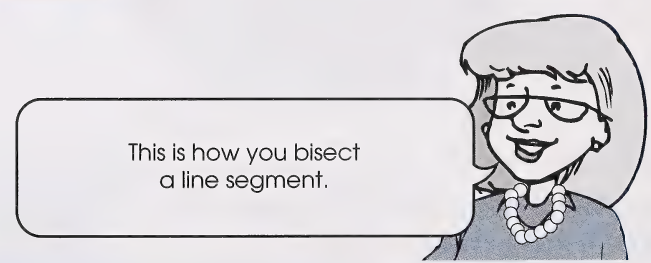
In most mathematical instrument sets there is a **compass**. You can construct special angles of 30°, 60°, 90°, and 45° with a compass and straightedge.



To construct these special angles you must be able to do the following constructions.

- **bisect** a line segment
- bisect an angle
- construct congruent line segments

To bisect means to divide a line segment or angle into two equal parts.



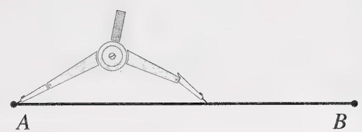
Example 1

Use a compass and straightedge to bisect $\overline{AB}$.

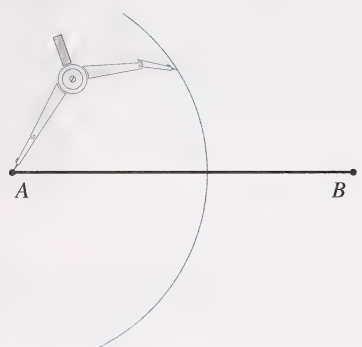
$\overline{AB}$ is read as "line segment AB."

Solution

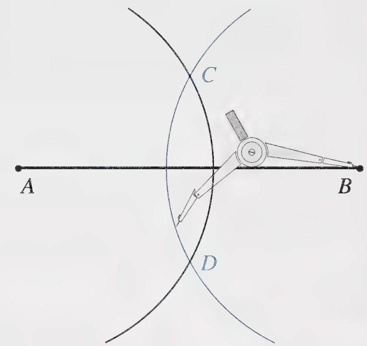
Step 1: Place the point of a compass on A. Open the compass to **more** than half the length of $\overline{AB}$.



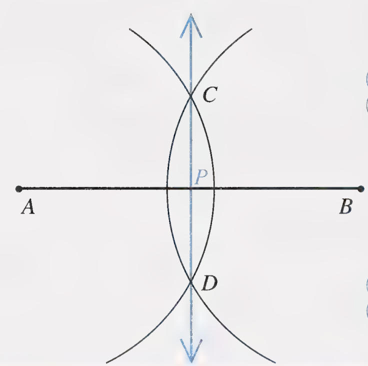
Step 2: Keep the compass setting the same and draw an arc cutting $\overline{AB}$. Ensure that the arc is about a semicircle.



Step 3: Without changing the setting on the compass, place the compass point at B. Then draw another arc that cuts the first arc at C and D.



Step 4: Use a straightedge to draw $\overleftrightarrow{CD}$. Label the point P where $\overleftrightarrow{CD}$ cuts $\overline{AB}$. ($\overleftrightarrow{CD}$ is the **perpendicular bisector** of $\overline{AB}$.) $\overline{AB}$ has been bisected. It has been divided into two congruent parts; $\overline{AP} \cong \overline{PB}$ and $\angle CPB = 90°$.

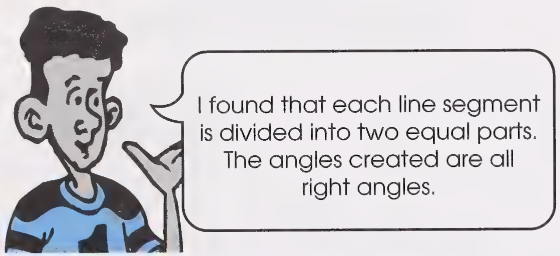


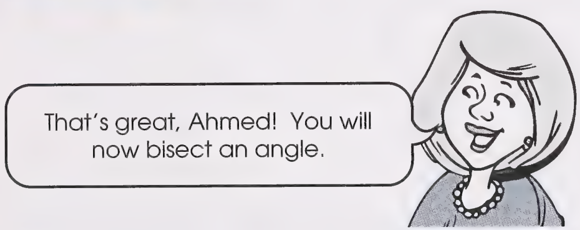
$\overleftrightarrow{CD}$ is read as "line CD."

$\cong$ is read as "is congruent to."

3. Draw several segments of different lengths on your paper. Then bisect each segment using only a compass and a straightedge.

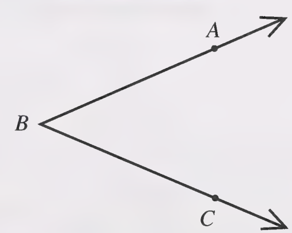
Check the accuracy of each construction with a ruler and a protractor.





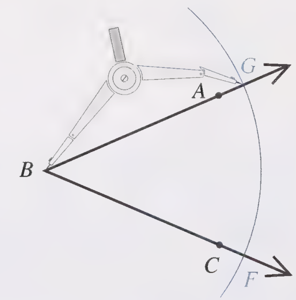
Example 2

Use a compass and straightedge to bisect $\angle ABC$.

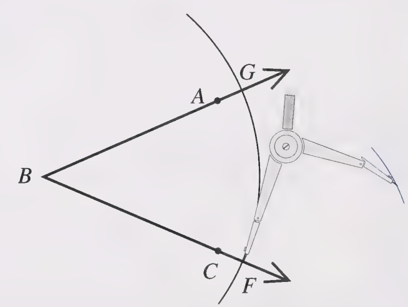


Solution

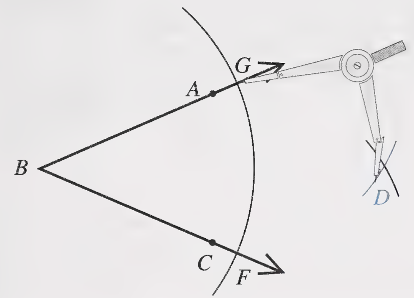
Step 1: Place the point of a compass on B. Draw an arc which intersects the rays of the angle at G and F. **Note:** Do not make the arc too close to the vertex of the angle. You may extend the rays of the angle if you wish.



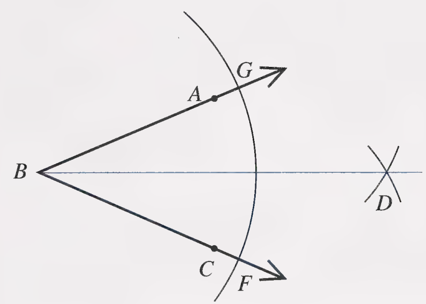
Step 2: Place the point of the compass on F and draw a second arc. **Note:** Make sure this arc extends at least halfway across the mouth of the angle.



Step 3: Without changing the setting of the compass, place the point of the compass on G and draw a third arc that intersects the second arc at D.



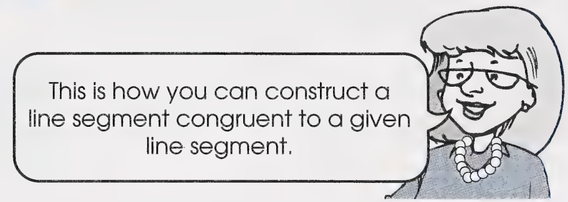
Step 4: Use a straightedge to draw $\overline{BD}$. ($\overline{BD}$ is the **angle bisector** of $\angle ABC$.)



$\angle ABC$ has been bisected. It has been divided into two congruent parts; $\angle ABD \cong \angle DBC$.

4. Draw several angles of different sizes on your paper. Then bisect each angle using only a compass and straightedge.

Check the accuracy of your constructions with a protractor. The two angles formed when each angle is bisected should be the same size.



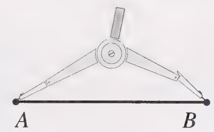
Example 3

Construct $\overline{XP}$ congruent to $\overline{AB}$.

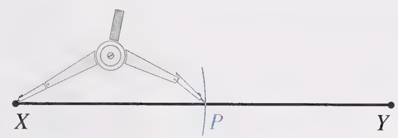
Solution

Step 1: Use a straightedge to draw $\overline{XY}$ on your paper. Do not measure the segment, but make it longer than $\overline{AB}$.

Step 2: Set a compass to the length of $\overline{AB}$.



Step 3: Without changing the compass setting, place the point of the compass on X, and draw an arc cutting $\overline{XY}$ at P.



$\overline{XP} \cong \overline{AB}$

5. Draw several segments of various lengths on your paper. Use a compass and straightedge to construct congruent segments.

Use a ruler to check the accuracy of your constructions. Each segment you construct should be the same length as the segment you copied.

Example 4

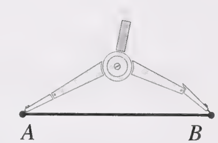
Construct an angle of 60°.

Solution

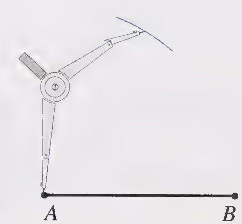
An equilateral triangle has three angles of 60°. So, if you construct an equilateral triangle, you will have an angle of 60°.

Step 1: Draw any segment and label it $\overline{AB}$.

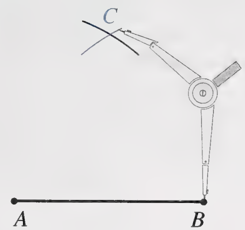
Step 2: Set a compass to the length of $\overline{AB}$.



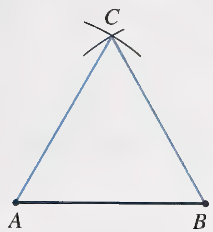
Step 3: Without changing the setting of the compass, place the point of the compass on A and make an arc above $\overline{AB}$.



Step 4: Keeping the same compass setting, place the point of the compass on B and draw an arc that intersects the first arc at C.



Step 5: Use a straightedge to draw $\overline{AC}$ and $\overline{BC}$.



$\angle CAB = 60°$, $\angle ACB = 60°$, and $\angle CBA = 60°$.

6. **a.** Use a compass and straightedge to construct an angle of 60°.

b. Use a compass and straightedge to construct an angle of 30°.
Hint: Construct an angle of 60° and bisect it.

Check your constructions using a protractor.

Earlier in this activity you bisected line segments. You discovered that the line which bisects a segment is a perpendicular bisector. Therefore, you can construct an angle of 90° by bisecting a line segment.

7. **a.** Use a compass and straightedge to construct an angle of 90°.

b. Use a compass and straightedge to construct an angle of 45°.
Hint: Construct an angle of 90° and bisect it.

Check your constructions using a protractor.

Did You Know?

Geometers distinguish between a geometric construction and a drawing. A geometric construction is done using only a compass and a straightedge. A drawing can be done with any tool.

The compass and straightedge are sometimes called Euclidian tools after the Greek mathematician Euclid who lived around 300 B.C.

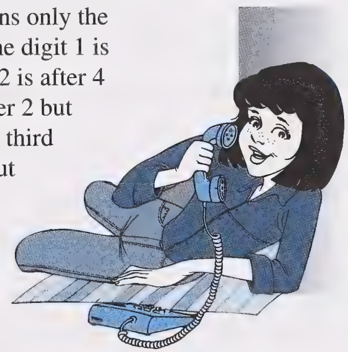
Use the Internet to discover more about Euclid.

Now Try This

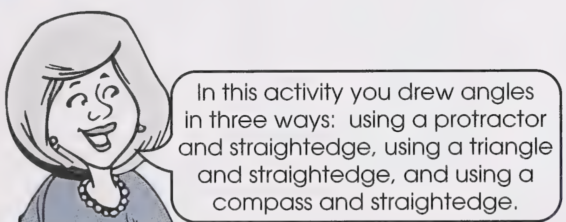
Use a problem-solving strategy to answer the following question.

8. Martha's phone number contains only the digits 1, 2, 3, 4, 5, 6, and 7. The digit 1 is before 3 but after 4. The digit 2 is after 4 but before 1. The digit 5 is after 2 but before 3. The digit 5 is not the third digit. The digit 7 is before 6 but after 3. The digit 6 is the last number.

What is Martha's phone number?



Check your answers by turning to the Appendix.



Follow-up Activities

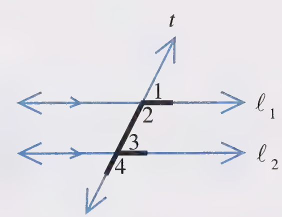
If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

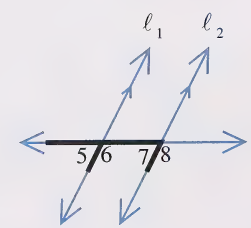
Extra Help

In this section you classified angles and lines. One of the classifications you explored was the angles formed by two parallel lines and a transversal.

If you had difficulty remembering these angles, these memory aids may be useful.

- Corresponding angles of parallel lines form a letter **F**. The letter F may not be in standard position.

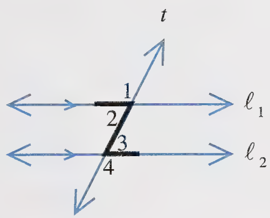


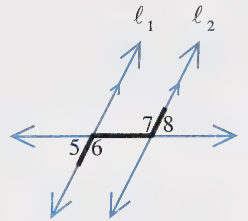


$\angle 1$ and $\angle 3$ are corresponding angles.
$\angle 2$ and $\angle 4$ are corresponding angles.
$\angle 5$ and $\angle 7$ are corresponding angles.
$\angle 6$ and $\angle 8$ are corresponding angles.

Note: Corresponding angles of parallel lines are equal.

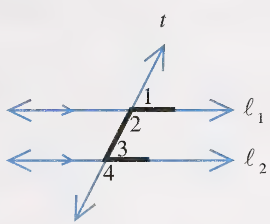
- Interior alternate and exterior alternate angles of parallel lines form a letter **Z**. The letter Z may not be in standard position.

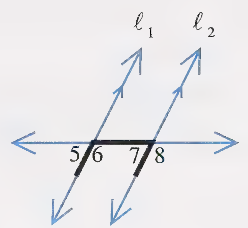




$\angle 1$ and $\angle 4$ are exterior alternate angles.
$\angle 2$ and $\angle 3$ are interior alternate angles.
$\angle 5$ and $\angle 8$ are exterior alternate angles.
$\angle 6$ and $\angle 7$ are interior alternate angles.

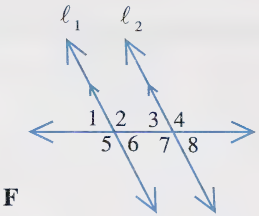
- Co-interior and co-exterior angles of parallel lines form a letter **C**. The letter C may not be in standard position.





$\angle 1$ and $\angle 4$ are co-exterior angles.
$\angle 2$ and $\angle 3$ are co-interior angles.
$\angle 5$ and $\angle 8$ are co-exterior angles.
$\angle 6$ and $\angle 7$ are co-interior angles.

1. In your notebook, make three diagrams similar to the given diagram. Use a coloured pen to draw the lines. Use a black pen to label the diagram.

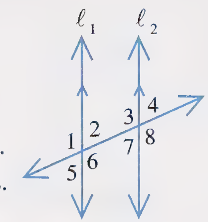


a. Use a black pen to show the letter **F** on the first diagram. Identify the corresponding angles.

b. Use a black pen to show the letter **Z** on the second diagram. Identify the interior alternate and exterior alternate angles.

c. Use a black pen to show the letter **C** on the third diagram. Identify the co-interior and co-exterior angles.

2. Each of the following statements refers to the given diagram. Copy the statements and fill in the blanks.



a. $\angle 6$ and $\angle 3$ are ______________ angles.
b. $\angle 5$ and $\angle 4$ are ______________ angles.
c. $\angle 5$ and $\angle 7$ are ______________ angles.
d. $\angle 6$ and $\angle 7$ are ______________ angles.
e. $\angle 5$ and $\angle 8$ are ______________ angles.

3. In this section, you classified angles and lines. You can use your knowledge to calculate unknown angles.

a. Find the puzzle "Daffynition Decoder" in the Appendix. Photocopy the page and complete the puzzle.

b. Find the puzzle “Why Couldn’t the Two Elephants Go Swimming Together?” in the Appendix. Photocopy the page and complete the puzzle.

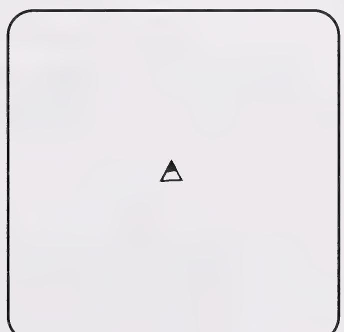
Check your answers by turning to the Appendix.

Enrichment

In this section you drew angles using different mathematical instruments. You can also draw angles using a computer and a drawing program.

One computer drawing program that many schools use is **Logo**.

When Logo is loaded into a computer, the **turtle** (a triangular shaped drawing tool) is active and appears in the turtle window. The turtle is in its **home** position—in the centre of the screen pointing in an upwards direction.



Example 1

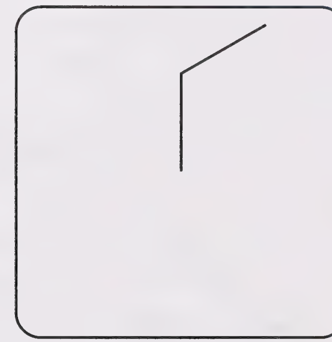
Use a computer and a Logo program to draw an angle of 120°.

Solution

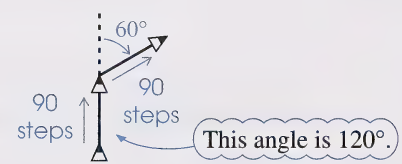
One way to draw an angle of 120° is to type the following series of commands. (Press “Return” after each command.)

```
FORWARD 90     or     FD 90
RIGHT 60              RT 60
FORWARD 90            FD 90
HIDE TURTLE           HT
```

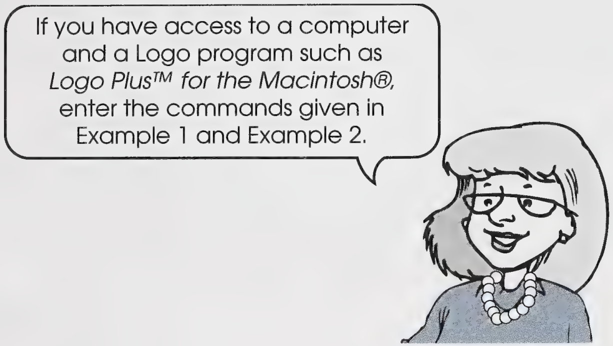
The screen looks like this; an angle of 120° is drawn.



Note: The turtle begins the drawing in its home position—in the centre of the screen pointing in an upward direction. The turtle moves forward 90 steps, turns right 60° from the direction it was moving, moves forward 90 steps, and then disappears.



Type **SHOWTURTLE** or **ST** and press “Return” to make the turtle appear again. Type **HOME** and press “Return” to make the turtle go to its home position. Lastly, type **CLEARSCREEN** or **CS** and press “Return” to clear the screen.



Example 2

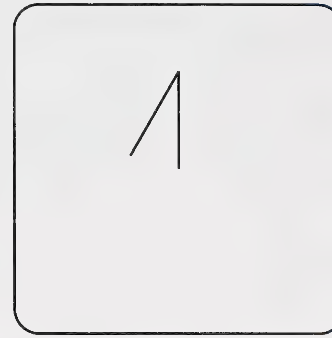
Use a computer and a Logo program to draw an angle of 30°.

Solution

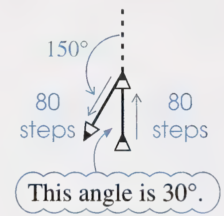
One way to draw an angle of 30° is to type the following series of commands. (Press “Return” after each command.)

```
FORWARD 80     or     FD 80
LEFT 150              LT 150
FORWARD 80            FD 80
HIDE TURTLE           HT
```

The screen looks like this; an angle of 30° is drawn.



Note: The turtle begins the drawing in its home position. The turtle moves forward 80 steps, turns left 150° from the direction it was moving, moves forward 80 steps, and disappears.



Type **SHOWTURTLE** or **ST** and press “Return” to make the turtle appear again. Type **HOME** and press “Return” to make the turtle go to its home position. Lastly, type **CLEARSCREEN** or **CS** and press “Return” to clear the screen.

1. Use a ruler and a protractor to make a diagram showing the turtle's movements for each of the following series of commands.

a. FD 40
RT 150
FD 60

b. FD 60
LT 20
FD 50

c. FD 40
RT 80
FD 75

d. FD 60
LT 110
FD 70

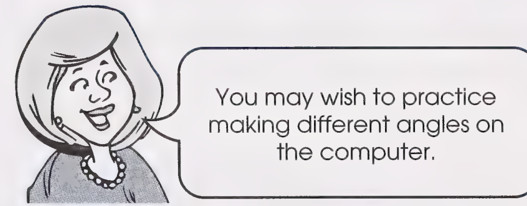
Check your answers by turning to the Appendix.

If you have access to a computer and a Logo program such as *Logo Plus™ for the Macintosh®*, do question 2.

2. Draw the following angles on the computer. **Note:** After each set of commands, type **HOME** and press "Return" to make the turtle go to its home position. Lastly, type **CLEARSCREEN** or **CS** and press "Return" to erase the previous drawing.

a. 90° **b.** 120° **c.** 145°

Check your answers by measuring the angles on the screen with a protractor.



Did You Know?

Dr. Seymour Papert, working with a group of scientists, created Logo during the late 1960s and early 1970s. Dr. Papert got the idea for this computer program after watching a computer direct a pen to draw a picture. The pen was mounted in an apparatus that looked like a turtle.

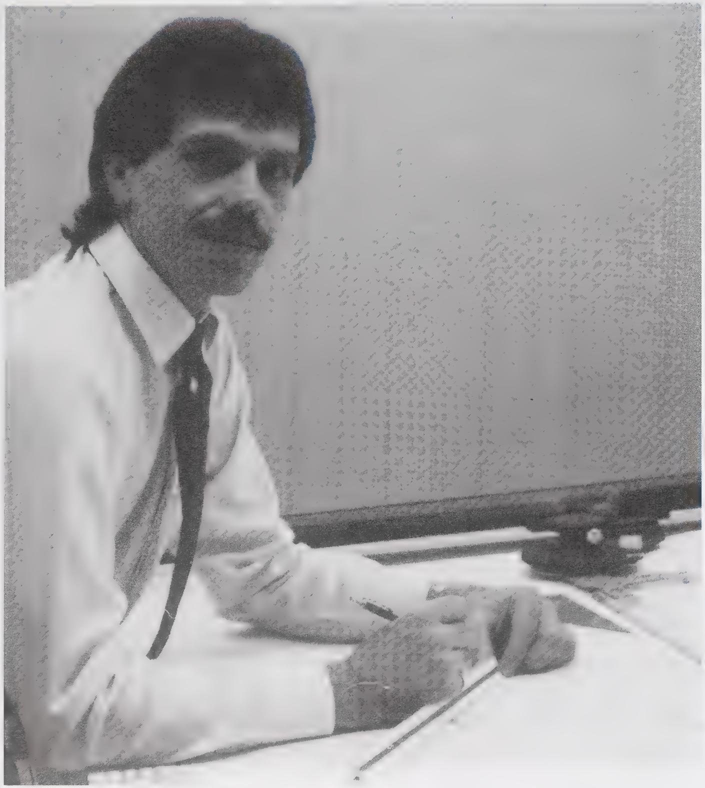
If you wish to know more about Dr. Papert and Logo programming, you may find this site interesting. It features questions and answers on Logo, a glossary, references, programs you can download, and more.

http://www.primenet.com/pcai/New_Home_Page/ai_info/pcai_logo.html

Now Try This

Visit a graphic artist and discover how computer programs such as *Adobe Illustrator™* are used to draw angles. Also, find out how slides, flips, and turns are performed on the computer.

Conclusion



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In this section you investigated lines and angles. You classified pairs of lines, individual angles, and pairs of angles. You drew angles with a variety of tools and instruments—a protractor and straightedge, a triangle and straightedge, and a compass and straightedge. If you did the Enrichment, you also drew angles using a computer.

Take a close look at your own home and the various buildings or structures in and around your community. How have the architects and engineers used lines and angles in the designs of these projects?

Assignment

Assignment Booklet

You are now ready to complete the module assignment for Section 3.

Module Summary

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In Module 6 you had opportunities to build things, use tools and instruments, and make designs. In your investigation of shape and space you used maps, globes, and many different kinds of measuring tools and instruments. You drew slide, flip, and turn images of figures. You tested figures for flip symmetry and turn symmetry. You made designs involving slides, flips, and/or turns. You used mathematical instruments (and perhaps a computer) to construct angles and lines. You sorted and classified angles and lines.

This old Chinese proverb emphasizes the importance of hands-on activities to learning.

I hear and I forget.
I see and I remember.
I do and I understand.

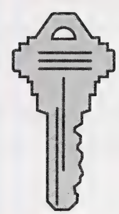
Do you learn better by doing?

Final Module Assignment

Assignment Booklet

You are now ready to complete the final module assignment.

APPENDIX



- Glossary
- Suggested Answers
- Map
- Puzzles

Glossary

Acute angle: an angle which is less than a quarter turn; an angle less than 90°

Adjacent angles: any two angles that are next to each other and share a common vertex and ray

Angle: a geometric figure formed by two distinct rays starting from the same endpoint

Bisect: divide a line segment or an angle into two equal parts

Capacity: the amount a container will hold

Co-exterior angles: (of parallel lines cut by a transversal) any two angles that are outside the parallel lines and on the same side of the transversal

Co-interior angles: (of parallel lines cut by a transversal) any two angles that are inside the parallel lines and on the same side of the transversal

Complementary angles: two angles that have a sum of 90°

Copy: make exactly the same; duplicate

Corresponding angles: (of parallel lines cut by a transversal) any two angles on the same side of the parallel lines and on the same side of the transversal

Daylight-saving time: a system for turning clocks ahead one hour in the spring to extend the daylight hours during the time when most people are awake

Exterior alternate angles: (of parallel lines cut by a transversal) any two angles that are outside the parallel lines and on opposite sides of the transversal

Flip: in geometry, a motion that occurs when an object is flipped over a line; also called a **reflection**

Flip image: the new position of a figure after a flip; also called the **reflection**

Flip line: the line over which a figure is flipped; the **line of reflection**

Flip symmetry: the property whereby one-half of a figure can be flipped onto the other half; the condition whereby one-half of a figure is the mirror image of the other half; also called **line symmetry** or **reflection symmetry**

Interior alternate angles: (of parallel lines cut by a transversal) any two angles that are inside the parallel lines and on the opposite sides of the transversal

Interior angles: angles inside a geometric figure

International Date Line: an imaginary line that runs north and south, mostly along 180° longitude

Intersecting lines: lines that cross

Line of symmetry: the line which divides a figure into two parts that are congruent (exactly the same size and shape)

Mass: the amount of matter in an object

Meridian: an imaginary line running around the earth from north to south along lines of longitude

Mira: a semitransparent plastic instrument used in geometry

Obtuse angle: an angle which is more than a quarter turn and less than a half turn (between 90° and 180°)

Order of turn symmetry: the number of times a figure fits on itself in a full turn

Parallel lines: lines that will never cross; symbol ‖

Perpendicular bisector: a bisector which cuts a line segment into two congruent parts, and meets the segment at a right angle

Perpendicular lines: intersecting lines which meet to form a right angle; symbol ⊥

Point of symmetry: the turn centre about which a figure with turn symmetry may be rotated

Prime: the symbol (′) placed above and to the right of a letter to show the new position of a point after a slide, flip, or turn

Prime meridian: the first meridian; the meridian which runs along 0° longitude through Greenwich, England

Problem: a task for which the method of finding the answer (as well as the answer) is not immediately known

Ray: a portion of a line starting at one point and going on forever

Reflex angle: an angle whose measure is more than half a turn and less than a full turn (between 180° and 360°)

Right angle: an angle whose measure is exactly a quarter turn (90°)

Right-angle triangle: a triangle that has an angle of 90°

Scientific notation: a way of writing a number as a number between 1 and 10, multiplied by a power of ten

Slide: in geometry, the movement of an object from its original position to a new position in a straight line; also called a **translation**

Slide arrow: the arrow which shows the direction and distance of a slide

Slide image: the new position of a figure after a slide

Solar time: system of keeping time where noon is the time when the sun is at its highest point in the sky

Standard form: the usual form of a number

Standard time system: a system of keeping time in which the world is divided into 24 time zones

Straight angle: an angle which is exactly a half turn (180°)

Supplementary angles: two angles that have a sum of 180°

Template: a stiff piece of material used as a pattern

Tessellation: an arrangement of congruent figures that covers a surface without gaps or overlapping

Transversal: a line which cuts across two or more parallel lines

Turn: in geometry, a motion that occurs when an object is turned around a fixed point; also called a **rotation**

Turn arrow: the arrow which indicates the direction of a turn and the angle of the turn

Turn centre: the point about which a figure is turned; also called the **point of rotation**

Turn image: the new position of a figure after a turn

Turn symmetry: the property that a figure coincides with its original position more than once in a full turn; also called **rotational symmetry** or **point symmetry**

Vertex: (of an angle) the endpoint from which the two rays start

Vertically opposite angles: (of two intersecting lines) any two nonadjacent angles; also called **vertical angles** and **opposite angles**

Volume: the amount of space an object occupies

Suggested Answers

Section 1: Activity 1

1. Answers will vary. The following answers are examples of acceptable responses.

a. The width of your little finger could be used to measure the length of a paper clip.

b. The width of your hand could be used to describe or measure the width of a desk.

c. The length of a person's arm could be used to describe or measure the height of a room.

2. Answers will vary. The following answers are examples of acceptable responses.

a. A stone's throw is the distance a person can throw a stone; it is a short distance.

b. Seven paces wide is the distance a person strides in seven steps or paces.

3. a. Answers will vary depending on the size of the people doing the measuring.

b. No, measurements will vary because people are different sizes. It would be better to use a standard measurement.

4. Answers will vary. Imperial units of length are still used today in football (yards), in horse racing (furlongs and miles), in the building trades (feet and inches), in measuring people's heights (feet and inches), and in printing (inches). You may find many instances where both metric and imperial measures are used.

5. **a.** In the chart each unit is 10 times the unit above it.

Metric Units of Length	Pattern
1 millimetre (mm) = 0.001 metre	
1 centimetre (cm) = 0.01 metre	×10
1 decimetre (dm) = 0.1 metre	×10
1 metre (m)	×10
1 decametre (dam) = 10 metres	×10
1 hectometre (hm) = 100 metres	×10
1 kilometre (km) = 1000 metres	×10

b. The metric system has a pattern, so it is easy to change from one unit to another. The imperial system does not have a pattern, so it is difficult to change from one unit to another.

c. Answers will vary. One reason is tradition. Canada used imperial units prior to 1970 and the United States continues to use imperial units. Therefore, traditions have been maintained in sports such as football and horse racing. Another reason is convenience. For example, land in western Canada was measured in imperial units, so it is easier to give distances between farm sections in miles.

6. **a.** centimetre or millimetre **b.** millimetre
c. centimetre or millimetre **d.** metre or centimetre

7. **a.** The paper clip is about 35 mm or 3.5 cm **or** the paper clip is about 34.5 mm or 3.45 cm.

b. The nail is about 58 mm or 5.8 cm **or** the nail is about 57.5 mm or 5.75 cm.

c. The tack is about 8 mm or 0.8 cm **or** the tack is about 7.5 mm or 0.75 cm.

d. The screw is about 30 mm or 3.0 cm **or** the screw is about 29.5 mm or 2.95 cm.

8. Answers will vary. The following answers are examples of acceptable responses.

a. A metric plastic, wooden, or metal ruler would be best for measuring the length, width, and thickness of a book.

b. A tailor's metric measuring tape would be best for measuring the circumference of a small bottle.

c. A carpenter's metric measuring tape would be best for measuring the length, width, and height of a room.

d. A micrometer would be best for measuring the diameter of a pen.

e. An odometer would be best for measuring the distance a car travels.

9. Estimates are checked by measuring.

10. Your answers may vary, as some items are not very specific.

a. kilogram or tonne **b.** kilogram
c. kilogram **d.** gram or kilogram
e. gram **f.** tonne or kilogram
g. gram **h.** tonne or kilogram
i. milligram

11. Estimates are checked by measuring.

12. Estimates are checked by measuring.

13. **a.** millilitre **b.** litre **c.** millilitre **d.** litre

14. The capacities are the same.

15. **a.** 350 mL = **350** cm^3 **b.** 15 mL = **15** cm^3

c. 2 L = **2000** cm^3 **d.** 3 L = **3000** cm^3

16. Estimates are checked by measuring.

Now Try This

17. You can use the guess, check, and revise strategy to solve this problem. Answers will vary. This is one solution.

$$(4 \div 2 - 1) \times 3 = 3$$

Section 1: Activity 2

1. **a.** $12\,756.28 = 1.275\,628 \times 10^6$

Earth has a diameter of $1.275\,628 \times 10^6$ km.

b. $1\,083\,230\,000\,000 = 1.083\,23 \times 10^{12}$

Earth has a volume of $1.083\,23 \times 10^{12}$ km^3.

c. $5\,976\,000\,000\,000\,000\,000\,000 = 5.976 \times 10^{21}$

Earth has a mass of 5.976×10^{21} t.

2. **a.** $2.00 \times 10^7 = 20\,000\,000$

The temperature at the centre of the Sun is about 20 000 000°C.

b. $1.496 \times 10^6 = 1\,496\,000$

Earth is 1 496 000 km from the Sun.

c. $4.00 \times 10^{13} = 40\,000\,000\,000\,000$

Proxima Centauri is about 40 000 000 000 000 km from Earth.

Now Try This

3. You can use the guess, check, and revise strategy to solve this problem.

There are only 6 coins and the coins are pennies, nickels, and dimes.

Guess 1: Are the coins 1 penny, 2 nickels, and 3 dimes?

No, the problem states that there are more nickels than dimes.

Guess 2: Are the coins 1 penny, 2 dimes, and 3 nickels?

Yes, the problem states there are more nickels than dimes, and more dimes than pennies.

1 penny has a value of 1¢.
2 dimes have a value of 20¢.
3 nickels have a value of 15¢.

So, the speaker has 36¢ in his pocket.

Section 1: Activity 3

1. **a.** $\frac{3}{4}$ turn **b.** $\frac{5}{8}$ turn **c.** $\frac{7}{8}$ turn **d.** $\frac{1}{2}$ turn

2. The word is FACE.

3. The factors of 360 are 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, and 360.

4. Answers may vary slightly; however, they should be close to the following.

a. $\angle ABC = 50°$ **b.** $\angle HIJ = 95°$
c. $\angle LMN = 20°$ **d.** $\angle WXY = 160°$
e. $\angle RST = 135°$ **f.** $\angle EFG = 90°$
g. $\angle OPQ = 60°$ **h.** $\angle UVW = 120°$

5. **a.** 20° **b.** 145° **c.** 40°

6. **a.** $\angle KGF = 90°$ **b.** $\angle GBF = 87°$ **c.** $\angle CHD = 19°$
d. $\angle CHE = 43°$ **e.** $\angle JHD = 110°$ **f.** $\angle AKE = 62°$

7. You can use logic to answer this question.

- The speaker's age is between 60 and 80; so you can eliminate the other ages.
- The speaker's age is an even number; so you can eliminate the odd ages.

  The speaker's age is 62, 64, 66, 68, 70, 72, 74, 76, or 78.
- The speaker's age is a number whose digits total 11.

  $7 + 4 = 11$

So, the speaker's age is 74.

Section 1: Activity 4

1. **a.** 09:00 **b.** 12:00 **c.** 15:15 **d.** 22:20

2. **a.** 7 A.M. **b.** 1:20 P.M. **c.** 4:40 P.M. **d.** 11 P.M.

3. **a.** Atlantic standard time is 3 h ahead of Mountain standard time, so it is 13:00 AST.

b. Central standard time is 1 h ahead of Mountain standard time, so it is 11:00 CST.

4. **a.** Mountain standard time is 3 h behind Atlantic standard time, so it is 19:00 MST.

b. Eastern standard time is 1 h behind Atlantic standard time, so it is 21:00 EST.

5. a. The time in Victoria is 8 h less than the time at the prime meridian, so it is 12:00 in Victoria.

   b. The time in Edmonton is 7 h less than the time at the prime meridian, so it is 13:00 in Edmonton.

   c. The time in Regina is 6 h less than the time at the prime meridian, so it is 14:00 in Regina.

   d. The time in Winnipeg is 6 h less than the time at the prime meridian, so it is 14:00 in Winnipeg.

   e. The time in Toronto is 5 h less than the time at the prime meridian, so it is 15:00 in Toronto.

   f. The time in Quebec City is 5 h less than the time at the prime meridian, so it is 15:00 in Quebec City.

   g. The time in Fredericton is 4 h less than the time at the prime meridian, so it is 16:00 in Fredericton.

   h. The time in Charlottetown is 4 h less than the time at the prime meridian, so it is 16:00 in Charlottetown.

   i. The time in Halifax is 4 h less than the time at the prime meridian, so it is 16:00 in Halifax.

   j. The time in Whitehorse is 8 h less than the time at the prime meridian, so it is 12:00 in Whitehorse.

   k. The time in Yellowknife is 7 h less than the time at the prime meridian, so it is 13:00 in Yellowknife.

   l. The time in St. John's is $3\frac{1}{2}$ h less than the time at the prime meridian, so it is 16:30 in St. John's.

6. a. The time in Melbourne, Australia, is 10 h more than the time at the prime meridian, so it is 16:00 h in Melbourne.

   b. The time in Lima, Peru, is 5 h less than the time at the prime meridian, so it is 01:00 in Lima.

   c. The time in Tokyo, Japan, is 9 h more than the time at the prime meridian, so it is 15:00 in Tokyo.

   d. The time in Cape Town, South Africa, is 2 h more than the time at the prime meridian, so it is 08:00.

   e. The time in Rome, Italy, is 1 h more than the time at the prime meridian, so it is 07:00 in Rome.

   f. The time in New York, U.S.A., is 5 h less than the time at the prime meridian, so it is 01:00 in New York.

7. a. The time in Argentina is 2 h more than the time in Ottawa, so it is 12:00 in Argentina.

   b. The time in Scotland is 5 h more than the time in Ottawa, so it is 15:00 in Scotland.

   c. The time in Italy is 6 h more than the time in Ottawa, so it is 16:00 in Italy.

   d. The time in China is 13 h more than the time in Ottawa, so it is 23:00 in China.

e. The time in Japan is 14 h more than the time in Ottawa, so it is 24:00 in Japan.

f. The time in South Africa is 7 h more than the time in Ottawa, so it is 17:00 in South Africa.

8. a. The time in Charlottetown is 2 h **more** than the time in Regina.

$$\begin{array}{r} 23:00 \\ +\quad 2 \\ \hline 25:00 \end{array} = 01:00$$

Regrouping is required; 25:00 on Monday is the same as 01:00 on Tuesday.

It is 01:00 on Tuesday in Charlottetown.

b. The time in Vancouver is 3 h **less** than the time in Ottawa.

$$\begin{array}{r} 01:00 \\ -\quad 3 \\ \hline \end{array} = \begin{array}{r} 25:00 \\ -\quad 3 \\ \hline 22:00 \end{array}$$

Regrouping is required; 01:00 on Thursday is the same as 25:00 on Wednesday.

It is 22:00 on Wednesday in Vancouver.

c. The time in Winnipeg is 15 h **less** than the time in Tokyo.

$$\begin{array}{r} 07:30 \\ -\quad 15 \\ \hline \end{array} = \begin{array}{r} 31:30 \\ -\quad 15 \\ \hline 16:30 \end{array}$$

Regrouping is required; 07:30 on Wednesday is the same as 31:30 on Tuesday.

It is 16:30 on Tuesday in Winnipeg.

d. The time in Melbourne is 18 h **more** than the time in Whitehorse.

$$\begin{array}{r} 18:15 \\ +\quad 18 \\ \hline 36:15 \end{array} = 12:15$$

Regrouping is required; 36:15 on Friday is the same as 12:15 on Saturday.

It is 12:15 on Saturday in Melbourne, Australia.

9. a. Because the map is a two-dimensional representation of the world, the date line is shown 180° to the east and 180° to the west of the prime meridian. However, the two lines are, in reality, the same line.

b. The date line is halfway around the world from the prime meridian. Looking at the globe from the top, the prime meridian, which is 0° longitude, and the date line, which runs along 180° longitude meet.

10. In figures 2, 3, and 4, it is Sunday east of the date line and Monday west of the date line.

a. When travellers going from east to west cross the date line, they must set the date ahead one day.

b. When travellers going from west to east cross the date line, they must set the date back one day.

Now Try This

11. You can use a pattern to solve this problem.

$$\begin{array}{ccccccccccc} 1, & & 5, & & 12, & & 22, & & 35, & & 51 \\ & +4 & & +7 & & +10 & & +13 & & +16 & \\ & & 3 & & 3 & & 3 & & 3 & & \end{array}$$

The next two pentagonal numbers are 35 and 51.

Section 1: Follow-up Activities

Extra Help

1. **a.** 03:00 or 3 A.M.
b. 06:00 or 6 A.M.
c. 11:00 or 11 A.M.
d. 11:00 or 11 A.M.
e. 19:00 or 7 P.M.
f. 22:00 or 10 P.M.

2. **a.** **Step 1:** Find Argentina on the chart and read the time difference.

The time difference is +4 h.

Step 2: Calculate the time and day in Argentina.

$$\begin{array}{r} 12:00 \\ +\quad 4 \\ \hline 16:00 \end{array}$$

It is 16:00 on Wednesday in Argentina.

b. **Step 1:** Find Brazil on the chart and read the time difference.

The time difference is +3 h.

Step 2: Calculate the time and day in Brazil.

$$\begin{array}{r} 12:00 \\ +\quad 3 \\ \hline 15:00 \end{array}$$

It is 15:00 on Wednesday in Brazil.

Note: Actually this is the time for any part of Brazil. As mentioned previously, the chart does not make allowances for more than one time zone in a country.

c. **Step 1:** Find Egypt on the chart and read the time difference.

The time difference is +9 h.

Step 2: Calculate the time and day in Egypt.

$$\begin{array}{r} 12:00 \\ +\quad 9 \\ \hline 21:00 \end{array}$$

It is 21:00 on Wednesday in Egypt.

d. Step 1: Find South Africa on the chart and read the time difference.

The time difference is +9 h.

Step 2: Find the time and day in South Africa.

$$\begin{array}{r} 12:00 \\ +\quad 9 \\ \hline 21:00 \end{array}$$

It is 21:00 on Wednesday in South Africa.

e. Step 1: Find Malaysia on the chart and read the time difference.

The time difference is +15 h.

Step 2: Calculate the time and day in Malaysia.

$$\begin{array}{r} 12:00 \\ +\ 15 \\ \hline 27:00 \end{array} = 03:00$$

Regrouping is required; 27:00 on Wednesday is the same as 03:00 on Thursday.

It is 03:00 on Thursday in Malaysia.

f. Step 1: Find Thailand on the chart and read the time difference.

The time difference is +14 h.

Step 2: Calculate the time and day in Thailand.

$$\begin{array}{r} 12:00 \\ +\ 14 \\ \hline 26:00 \end{array} = 02:00$$

Regrouping is required; 26:00 on Wednesday is the same as 02:00 on Thursday.

It is 02:00 on Thursday in Thailand.

Enrichment

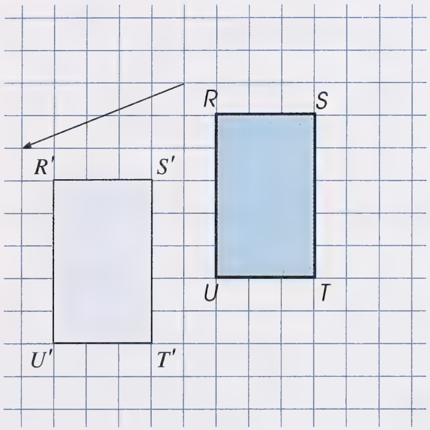
1. **a.** 10:00 **b.** 11:00 **c.** 11:00 **d.** 12:00
2. **a.** 10:00 **b.** 10:00 **c.** 11:00 **d.** 12:00

Note: Recall, Saskatchewan uses only standard time.

Section 2: Activity 1

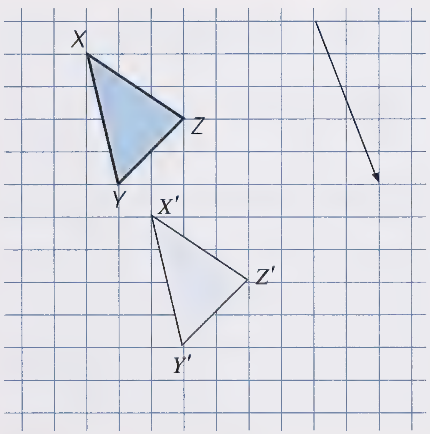
1. **a.** (R4, D4) **b.** (R5, U2)
   c. (L5, D3) **d.** (L5, D0) **or** (L5, U0)
   e. (R0, U4) **or** (L0, U4) **f.** (L3, U5)

2. a.



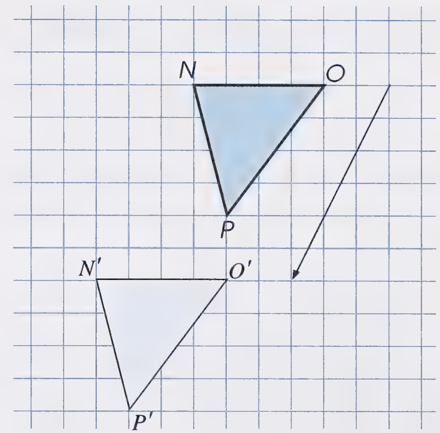
The slide rule is (L5, D2).

b.



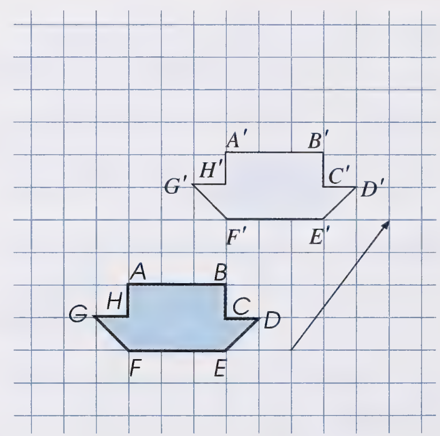
The slide rule is (R2, D5).

c.



The slide rule is (L3, D6).

d.



The slide rule is (R3, U4).

3. **a.** $(L5, U4)$ **b.** $(R0, D4)$ **or** $(L0, D4)$

Now Try This

4. You can use the divisibility tests (see Module 1) and logic to solve this problem.

   - If the number is divisible by 72, it is divisible by 8 and 9.
   - If the number is divisible by 8, the last digit must be even. So, the last digit is not 1, 3, 5, 7, or 9.
   - If the number is divisible by 8, the last 3 digits must be divisible by 8. 732, 734, 738 are not divisible by 8, but 736 is divisible by 8. So, the last digit is 6.

   42 73 **6**

   - If the number is divisible by 9, the sum of the digits is divisible by 9.

   $1+4+2+7+3+6=23$; 23 is not divisible by 9.
   $2+4+2+7+3+6=24$; 24 is not divisible by 9.
   $3+4+2+7+3+6=25$; 25 is not divisible by 9.
   $4+4+2+7+3+6=26$; 26 is not divisible by 9.
   $5+4+2+7+3+6=27$; 27 is divisible by 9.

   So, the first digit is 5 and the last digit is 6.

   5 42 73 **6**

   Note: You can also use the strategy of guessing, checking, and revising to solve the problem. You can use a calculator to check your guesses.

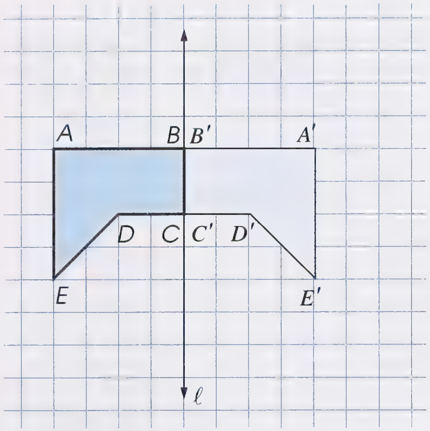
5. You can use an organized list to solve this problem.

Number of First Place	Number of Second Place	Number of Third Place	Total Points
3	0	0	15
2	1	0	13
2	0	1	11
1	2	0	11
1	1	1	9
1	0	2	7
0	3	0	9
0	2	1	7
0	1	2	5
0	0	3	3

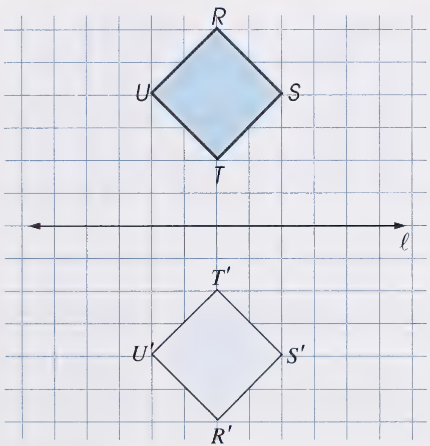
So, the smallest number of points a runner must earn in the three races to be guaranteed of winning more points than any other runner is 13. The runner will need two first places and one second place.

Section 2: Activity 2

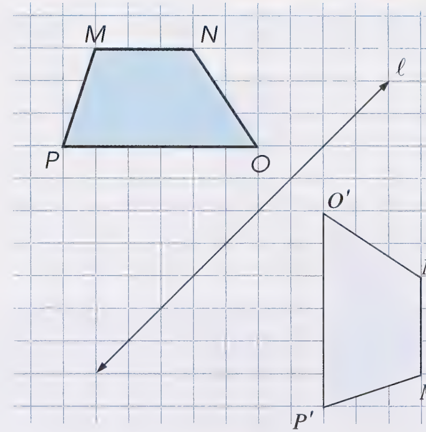
1. a.



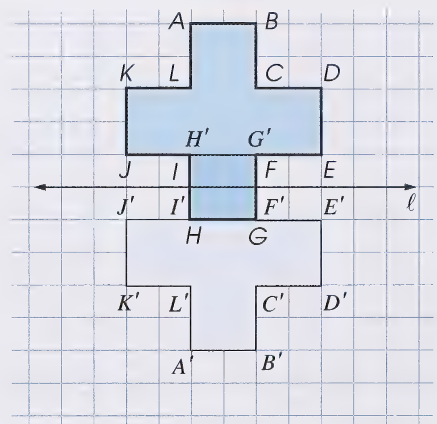
b.



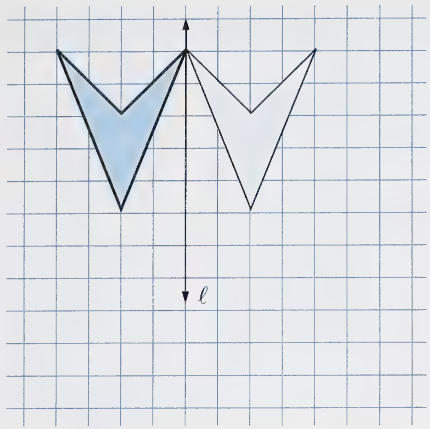
c.



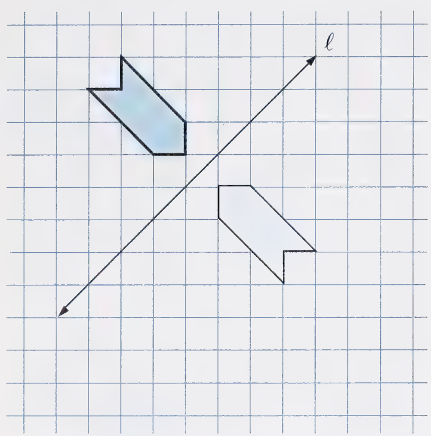
d.



2. a.



b.



Now Try This

3. You can use logic to help you solve this problem.

If one piece of string is three times the length of the other piece of string, there are four equal parts. Fold the string in half and then in half again. Cut the string through one fold. This will give you two pieces of string, one piece of which is three times the length of the other. (**Note:** You can check your answer by following these steps and then measuring the two lengths.)

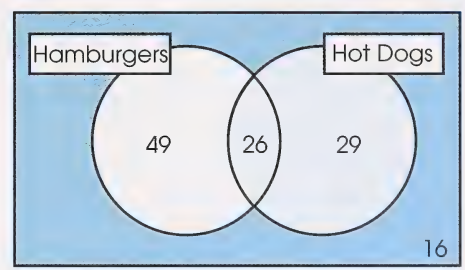
4. You can use a Venn diagram to solve this problem.

$$75 - 26 = 49$$

So, 49 people ate only hamburgers.

$$\begin{aligned} &120 - (49 + 26 + 16) \\ &= 120 - 91 \\ &= 29 \end{aligned}$$

So, 29 people ate only hot dogs.

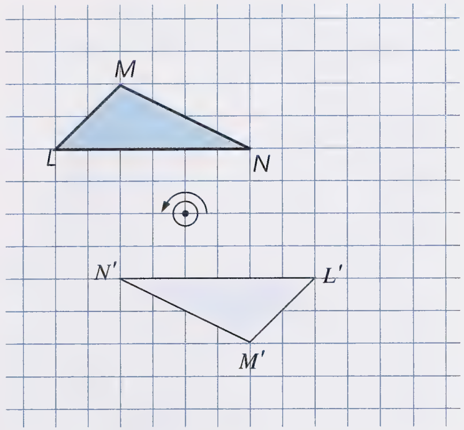


$29 + 26 = 55$

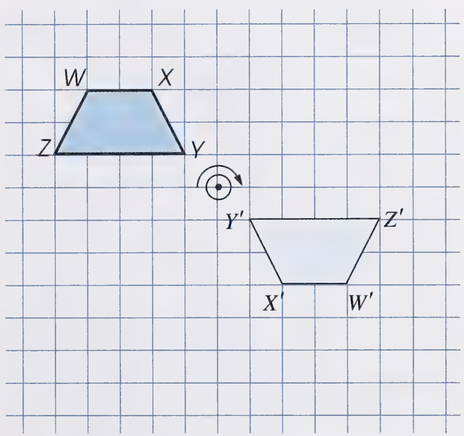
So, the total number of people who ate hot dogs was 55.

Section 2: Activity 3

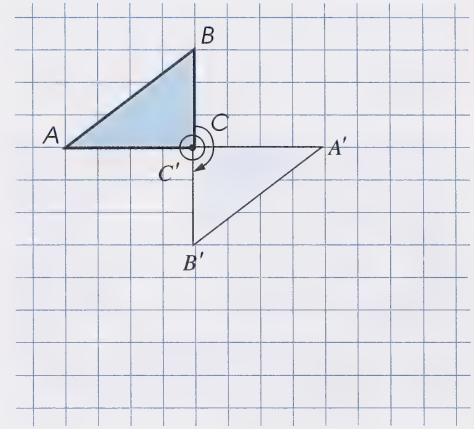
1. a.



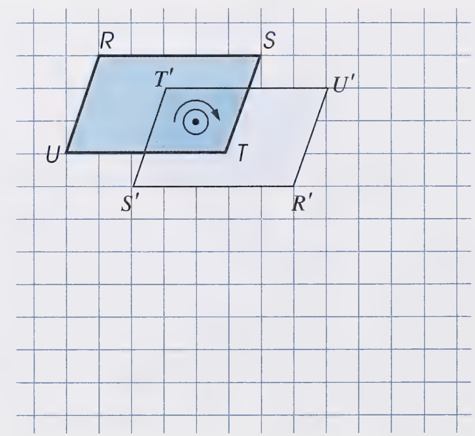
b.



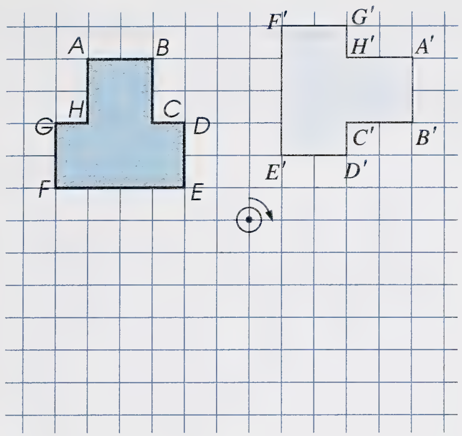
c.



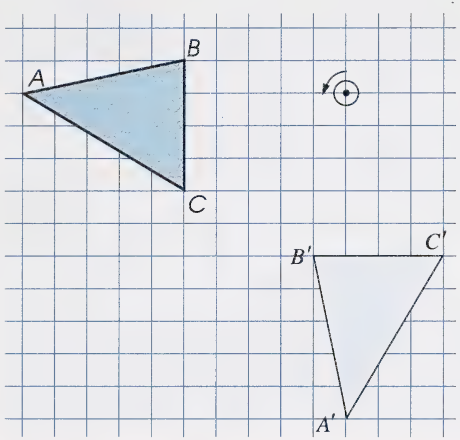
d.



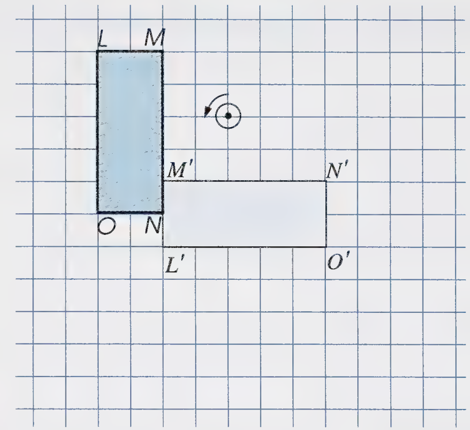
2. a.



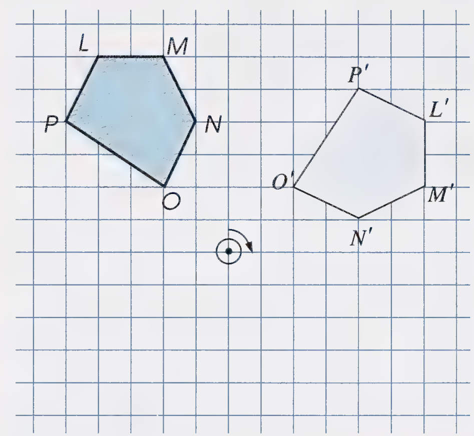
b.



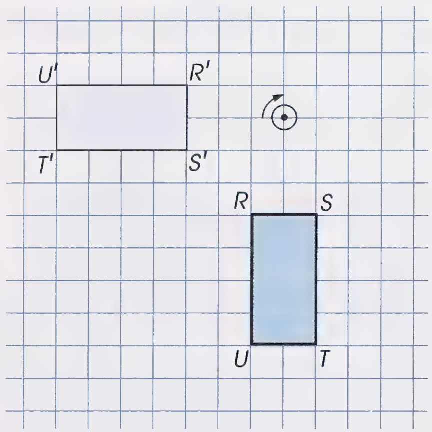
c.



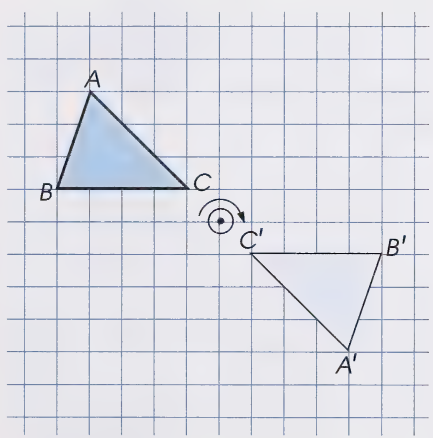
d.



3. **a.**



b.



4. Use a pattern to complete the table. $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$, $7^2 = 49$, $8^2 = 64$, $9^2 = 81$, $10^2 = 100$.

1	4	9	16	25	36	49
4	9	16	25	36	49	64
9	16	25	36	49	64	81
16	25	36	49	64	81	100

Section 2: Activity 4

1. **a.** no
   b. yes; 1 line of symmetry
   c. yes; 1 line of symmetry
   d. yes; 1 line of symmetry
   e. yes; 2 lines of symmetry
   f. no

2. The photographs were shared with your learning facilitator.

3. **a.** no
   b. yes; turn order of 4
   c. yes; turn order of 2
   d. yes; turn order of 6

4. **a.** no
   b. yes
   c. no
   d. no
   e. no
   f. yes
   g. no
   h. yes
   i. yes

5. The photographs were shared with your learning facilitator.

Now Try This

6. You can use the guess, check, and revise strategy and your knowledge of divisibility tests to solve this problem.

   The number is 59.

   Check: $59 < 80$

   $59 \div 2 = 29\ R1$ $59 \div 4 = 14\ R3$ $59 \div 6 = 9\ R5$

   $59 \div 3 = 19\ R2$ $59 \div 5 = 11\ R4$

Section 2: Activity 5

1. The design was shared with your learning facilitator.

2. The design was shared with your learning facilitator.

Now Try This

3. You can make an organized list to solve this question.

   The hands point in the same direction at or shortly after the following times.

   00:00, 01:05, 02:10, 03:15, 04:20, 05:25, 06:30, 07:35, 08:40, 09:45, 10:50, 11:55, 12:00, 13:05, 14:10, 15:15, 16:20, 17:25, 18:30, 19:35, 20:40, 21:45, 22:50, 23:55

   The hands point in the same direction 24 times in one day.

Section 2: Follow-up Activities

Extra Help

1. **a.** yes; 1 **b.** no **c.** no
   d. yes; 1 **e.** yes; 1 **f.** no
   g. yes; 1 **h.** yes; 2

2. **a.** yes

   b. no (The design appears balanced, but the head is turned to one side.)

   c. no (The design appears balanced, but the animal in the centre is turned to one side.)

   d. no (The design appears balanced, but the head is turned to one side and one wing has more feathers than the other.)

   e. no (The design appears balanced, but the left side does not exactly match the right side.)

   f. no

3. **a.** yes; 2 **b.** yes; 1 **c.** no **d.** no

   e. yes; 2

   f. no (The flag appears balanced, but because of the diagonal stripes, the top half does not exactly match the lower half, and the right half does not exactly match the left half.)

Enrichment

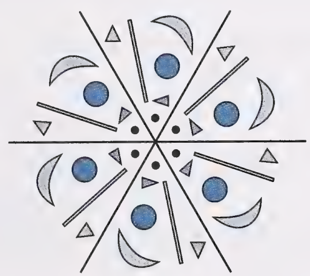
1. a. The angle is about 120°.

   b. The angle is about 90°.

   c. The angle is about 72°.

   d. The size of the angle decreases as the number of sides of the figure increases.

2. Your sketch should look similar to the following diagram.



Section 3: Activity 1

1. a. obtuse angle b. right angle c. acute angle
   d. straight angle e. acute angle f. right angle

2. a. $\angle CED$ and $\angle DEF$ are **complementary** angles.
   b. $\angle AEB$ and $\angle AEC$ are **supplementary** angles.
   c. $\angle RVS$ and $\angle TVU$ are **complementary** angles.
   d. $\angle YVZ$ and $\angle WVX$ are **supplementary** angles.

3. a.

Statement	Reason
$x+75°=90°$	The sum of complementary angles is 90°.
$\therefore\ x=15°$	

b.

Statement	Reason
$30°+50°+r=90°$	The sum of complementary angles is 90°.
$80°+r=90°$	
$\therefore\ r=10°$	

4. a.

Statement	Reason
$45°+58°+b=180°$	The sum of supplementary angles is 180°.
$103°+b=180°$	
$\therefore\ b=77°$	

b.

Statement	Reason
$m+65°=180°$	The sum of supplementary angles is 180°.
$\therefore\ m=115°$	

5. Yes, the three angles of each triangle from a straight angle.

6. a.

Statement	Reason
$c+23°+120°=180°$	The sum of the angles of a triangle is 180°.
$c+143°=180°$	
$\therefore\ c=37°$	

b.

Statement	Reason
$w+90°+45°=180°$	The sum of the angles of a triangle is 180°.
$w+135°=180°$	
$\therefore\ w=45°$	

c.

Statement	Reason
$r+45°+63°=180°$	The sum of the angles of a triangle is 180°.
$r+120°=180°$	
$\therefore\ r=72°$	

d.

Statement	Reason
$x+135°+25°=180°$	The sum of the angles of a triangle is 180°.
$x+160°=180°$	
$\therefore\ x=20°$	

7. **a.** yes **b.** no **c.** yes
d. yes **e.** yes **f.** no

8. **a.** yes **b.** no **c.** no
d. no **e.** yes **f.** no

9. **a.** yes **b.** yes **c.** yes **d.** yes

Note: The arrangements of the lines make them appear not to be parallel. These are optical illusions.

10. **a.** $\angle 2$ and $\angle 3$ are **adjacent** angles.
b. $\angle 2$ and $\angle 4$ are **vertically opposite** angles.
c. $\angle 3$ and $\angle 4$ are **adjacent** angles.
d. $\angle 3$ and $\angle 1$ are **vertically opposite** angles.

11. **a.** $\angle AXW$ and $\angle BXY$ are **vertically opposite** angles.
b. $\angle WXB$ and $\angle BXY$ are **adjacent** angles.
c. $\angle WXB$ and $\angle AXY$ are **vertically opposite** angles.
d. $\angle WXA$ and $\angle AXY$ are **adjacent** angles.

12. Vertically opposite angles are always equal.

13. **a.**

Statement	Reason
$y+70°=180°$ $\therefore\ y=110°$	The sum of supplementary angles is 180°.
$x=70°$	Vertically opposite angles are equal.
$z+70°=180°$ $\therefore\ z=110°$	The sum of supplementary angles is 180°.

So, $x=70°$, $y=110°$, and $z=110°$.

b.

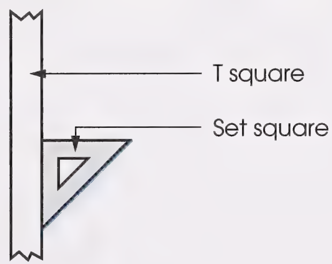
Statement	Reason
$w+50°=180°$ $\therefore\ w=130°$	The sum of supplementary angles is 180°.
$y=50°$	Vertically opposite angles are equal.
$x+50°=180°$ $\therefore\ x=130°$	The sum of supplementary angles is 180°.

So, $w=130°$, $x=130°$, and $y=50°$.

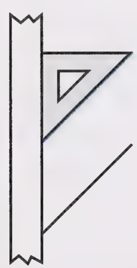
14. a. $\angle 5$ and $\angle 7$ are **corresponding** angles.
   b. $\angle 5$ and $\angle 4$ are **exterior alternate** angles.
   c. $\angle 5$ and $\angle 8$ are **co-exterior** angles.
   d. $\angle 2$ and $\angle 3$ are **co-interior** angles.
   e. $\angle 2$ and $\angle 7$ are **interior alternate** angles.
   f. $\angle 2$ and $\angle 4$ are **corresponding** angles.

15. Professionals such as graphic artists, architects, and engineers may use a T square and a set square (triangle) to draw parallel line segments. Here are the steps they use.

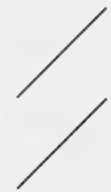
Step 1: Place a set square against a T square and use the edge of the set square to draw a line segment.



Step 2: Without moving the T square, slide the set square along the T square. Then draw a second line segment.



Step 3: Remove the T square and the set square.



The line segments are parallel.

16. a.

Statement	Reason
$\ell_1 \parallel \ell_2$	given
$y = 120°$	Corresponding angles of parallel lines are equal.
$y = z$ $\therefore\ z = 120°$	Vertically opposite angles are equal.
$y + x = 180°$ $120 + x = 180°$ $\therefore\ x = 60°$	The sum of supplementary angles is 180°.

So, $x = 60°$, $y = 120°$, and $z = 120°$.

b.

Statement	Reason
$\ell_1 \parallel \ell_2$	given
$b = 115°$	Corresponding angles of parallel lines are equal.
$b = d$ $\therefore\ d = 115°$	Vertically opposite angles are equal.
$c + d = 180°$ $c + 115 = 180°$ $\therefore\ c = 65°$	The sum of supplementary angles is 180°.

So, $b = 115°$, $c = 65°$, and $d = 115°$.

c.

Statement	Reason
$\ell_1 \parallel \ell_2$	given
$x = 70°$	Corresponding angles of parallel lines are equal.
$y = 70°$	Vertically opposite angles are equal.
$w + y = 180°$ $w + 70° = 180°$ $\therefore\ w = 110°$	The sum of supplementary angles is 180°.

So, $w = 110°$, $x = 70°$, and $y = 70°$.

Now Try This

17. You can solve this problem by making an organized list of all the page numbers containing the digit 1.

1, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 31, 41, 51, 61, 71, 81, 91, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125

The digit 1 occurs once in 1, 10, 12, 13, 14, 15, 16, 17, 18, 19, 21, 31, 41, 51, 61, 71, 81, 91, 100, 102, 103, 104, 105, 106, 107, 108, 109, 120, 122, 123, 124, and 125. The total is 32 times.

The digit 1 occurs twice in 11, 101, 110, 112, 113, 114, 115, 116, 117, 118, 119, and 121. The total is 24 times.

The digit 1 occurs three times in 111. The total is 3 times.

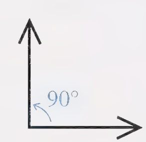
$$32 + 24 + 3 = 59$$

So, the digit 1 appears 59 times in the page numbers.

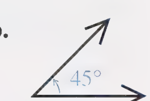
Section 3: Activity 2

1. Your angles should be similar to the following.

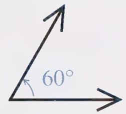
a.

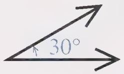


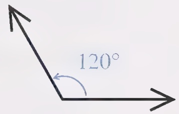
b.



c. 

d. 

e. 

f. 

2.–7. Note: The answers to questions 2 to 7 were checked by measuring with a protractor and/or ruler.

Now Try This

8. You can use logic to solve this problem.

Step 1: The last digit is given; it is 6.

_ _ _ - _ _ _ 6

Step 2: Read the statements again to decide what the first digit is. The choices are 1, 2, 3, 4, 5, and 7 (6 has been eliminated in Step 1). From the statements, you can eliminate 1, 2, 3, 5, and 7 as the first digit. So, the first digit is 4.

4 _ _ - _ _ _ 6

Step 3: Read the statements again to decide what the second digit is. The choices are 1, 2, 3, 5, and 7 (4 and 6 have been eliminated in previous steps). From the statements, you can eliminate 1, 3, 5, and 7. So, the second digit is 2.

4 2 _ - _ _ _ 6

Step 4: Read the statements again to decide what the third digit is. The choices are 1, 3, 5, and 7 (2, 4, and 6 have been eliminated in previous steps). From the statements you can eliminate 3, 5, and 7. So, the third digit is 1.

4 2 1 - _ _ _ 6

Step 5: Read the statements again to decide what the fourth digit is. The choices are 3, 5, and 7 (1, 2, 4, and 6 have been eliminated in previous steps). From the statements you can eliminate 3 and 7. So, the fourth digit is 5.

4 2 1 - 5 _ _ 6

Step 6: Read the statements again to decide what the fifth digit is. The choices are 3 and 7 (all other numbers have been eliminated in previous steps). From the statements you can eliminate 7.

4 2 1 - 5 3 _ 6

Step 7: There is only one digit left, so it must be the sixth digit.

4	2	1	-	5	3	7	6

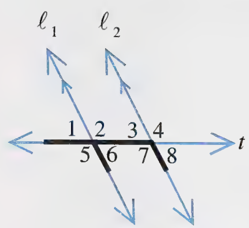
Martha's phone number is 421-5376.

Section 3: Follow-up Activities

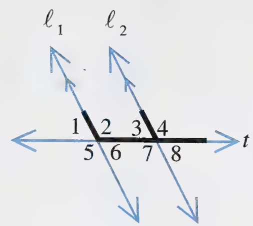
Extra Help

1. You can locate the letter **F** in two positions.

a.

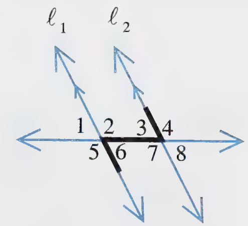


or

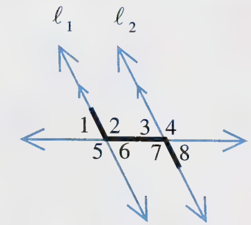


The corresponding angles are $\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$, $\angle 5$ and $\angle 7$, and $\angle 6$ and $\angle 8$.

b. You can locate the letter **Z** in two positions.

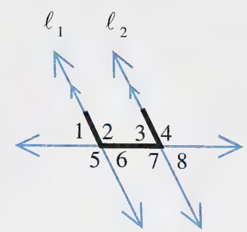


or

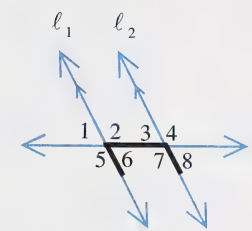


The interior alternate angles are $\angle 3$ and $\angle 6$, and $\angle 2$ and $\angle 7$. The exterior alternate angles are $\angle 4$ and $\angle 5$, and $\angle 1$ and $\angle 8$.

c. You can locate the letter **C** in two positions.



or



The co-interior angles are $\angle 2$ and $\angle 3$, and $\angle 6$ and $\angle 7$. The co-exterior angles are $\angle 1$ and $\angle 4$, and $\angle 5$ and $\angle 8$.

2. **a.** $\angle 3$ and $\angle 6$ are **interior alternate** angles.
   b. $\angle 4$ and $\angle 5$ are **exterior alternate** angles.
   c. $\angle 5$ and $\angle 7$ are **corresponding** angles.
   d. $\angle 6$ and $\angle 7$ are **co-interior** angles.
   e. $\angle 5$ and $\angle 8$ are **co-exterior** angles.

3. **a.** The daffynitions (definitions) are as follows:

Warehouse (Where… is the… house?):
WHAT•YOU•SAY•WHEN•LOST

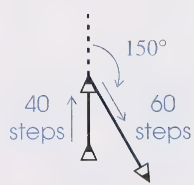
Explain (Eggs—plain):
NOT•SCRAMBLED•OR•FRIED

b. Why couldn't the two elephants go swimming together?

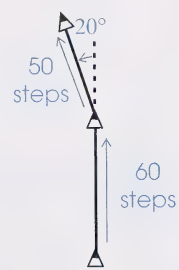
THEY•HAD•ONLY•ONE•PAIR•OF•TRUNKS

Enrichment

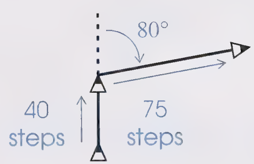
1. **a.**



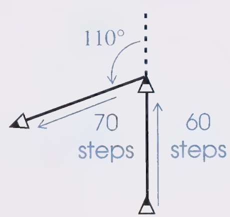
b.



c.

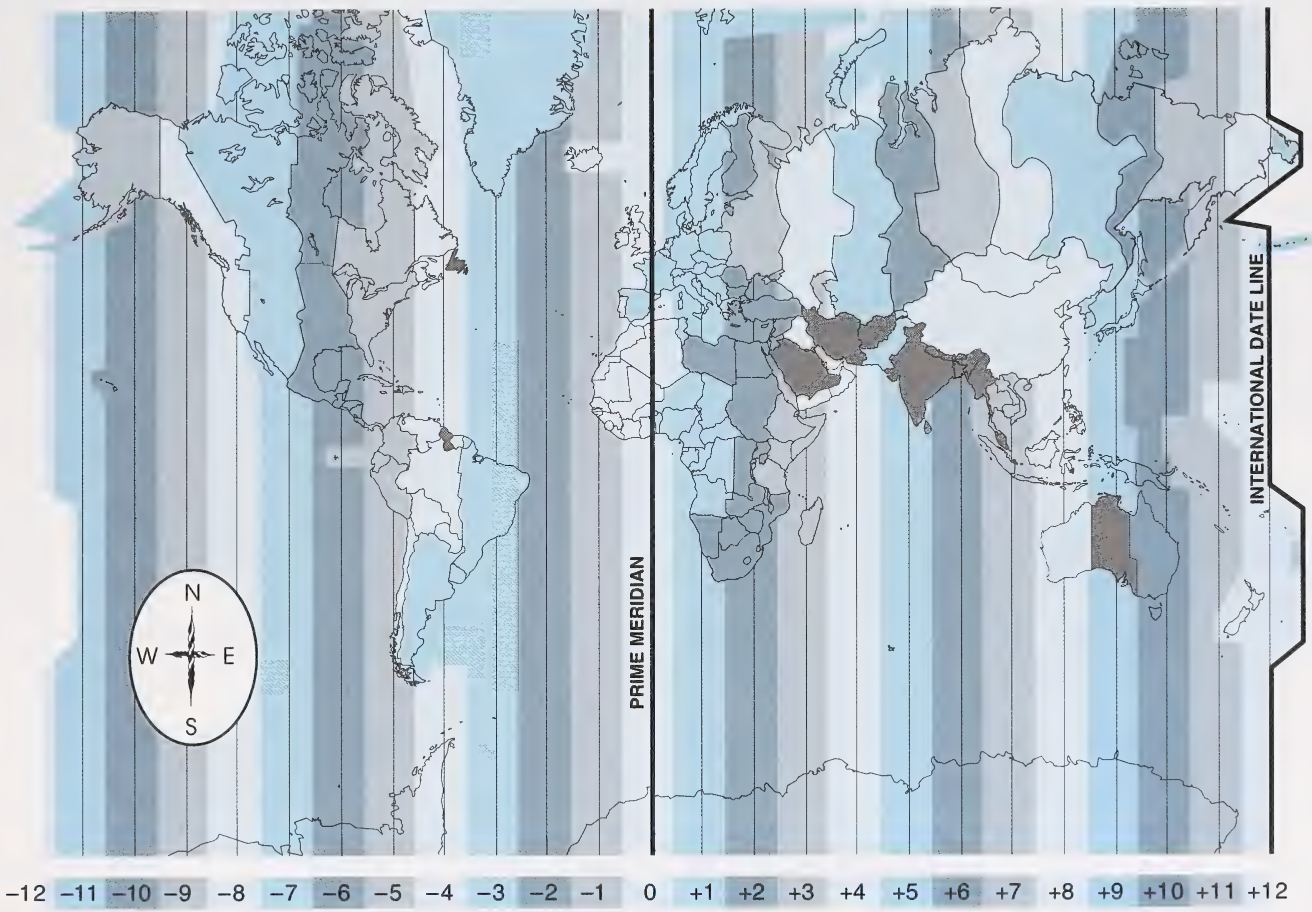


d.



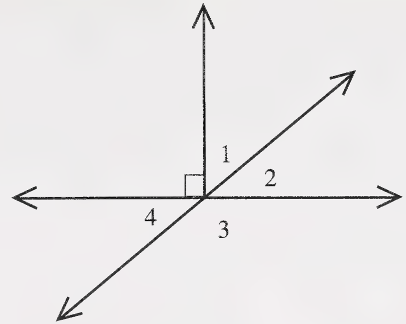
2. Check your computer diagrams with a protractor.

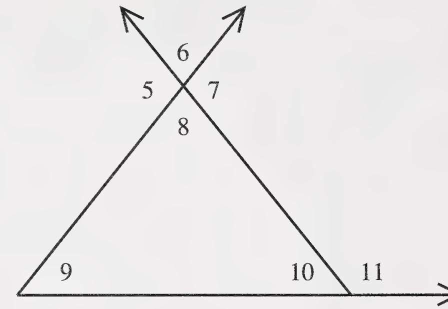
International Standard Time Zones



Daffynition Decoder[1]

For each statement, find the angle measure indicated. Look for each answer in the code. Each time the answer appears, write the letter of the statement above it.





Warehouse:

105°	40°	36°	78°	151°	55°	45°	146°	36°	151°	105°	40°	135°	42°	34°	55°	146°	78°

Explain:

42°	55°	78°	146°	116°	56°	36°	74°	29°	34°	135°	100°	55°	56°	60°	56°	98°	135°	100°

H If $\angle 1 = 50°$, then $\angle 2 =$

F If $\angle 3 = 120°$, then $\angle 4 =$

O If $\angle 2 = 35°$, then $\angle 1 =$

E If $\angle 4 = 45°$, then $\angle 3 =$

B If $\angle 6 = 29°$, then $\angle 8 =$

Y If $\angle 6 = 29°$, then $\angle 5 =$

C If $\angle 5 = 116°$, then $\angle 7 =$

I If $\angle 8 = 82°$, then $\angle 7 =$

A If $\angle 11 = 144°$, then $\angle 10 =$

N If $\angle 8 = 78°$ and $\angle 9 = 60°$, then $\angle 10 =$

D If $\angle 9 = 47°$ and $\angle 10 = 33°$, then $\angle 8 =$

U If $\angle 10 = 45°$ and $\angle 8 = 90°$, then $\angle 9 =$

M If $\angle 6 = 66°$ and $\angle 9 = 40°$, then $\angle 10 =$

T If $\angle 11 = 130°$ and $\angle 9 = 52°$, then $\angle 8 =$

W If $\angle 8 = 81°$ and $\angle 9 = 24°$, then $\angle 11 =$

R If $\angle 2 = 56°$, then $\angle 4 =$

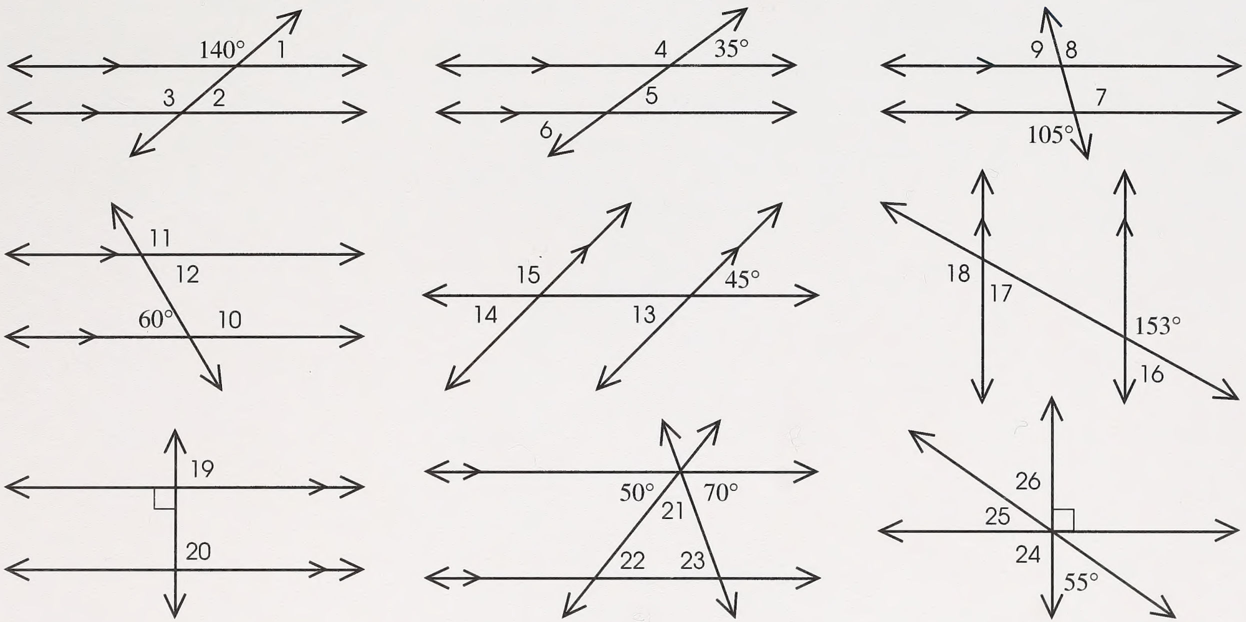
L If $\angle 1 = 56°$, then $\angle 4 =$

S If $\angle 1 = 56°$, then $\angle 3 =$

[1] 1989 Creative Publications for the puzzle from *Middle School Math with Pizzazz! Book D*.

Why Couldn't the Two Elephants Go Swimming Together?[1]

Give the measure of each numbered angle. Find your answer in the Code Key and notice the letter next to it. Write this letter in the box containing the number of the angle.



CODE KEY	
27°	A
35°	O
40°	R
45°	Y
50°	I
55°	P
60°	T
70°	U
75°	F
90°	N
105°	H
120°	E
135°	K
140°	L
145°	S
153°	D

12	7	10	14	8	16	18	6	20	3	13	25	19	11	26	17	22	1	5	9	21	2	23	24	15	4

[1] 1989 Creative Publications for the puzzle from *Middle School Math with Pizzazz! Book D.*

Mathematics 7
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